QUADRILATERALS - CHAPTER 10

EXERCISE 10A

Answer 1:

Given: Three angles of a quadrilateral are 75°, 90° and 75°.
Let the fourth angle be y.
Using angle sum property of quadrilateral,
$75°+90°+75°+y=360°$

$\Rightarrow 240°+y=360°$

$\Rightarrow y=360°-240°$

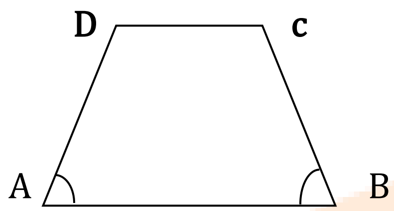
$\Rightarrow y=120°$
So, the measure of the fourth angle is 120°

.

Answer 2:

Let $\angle A = 2y^o$.
Then $\angle B = (4y)^o$; $\angle C = (5y)^o$ and $\angle D = (7y)^o$
Since the sum of the angles of a quadrilateral is 360^o, as ,
$2y + 4y + 5y + 7y = 360^o$
$\Rightarrow 18\ y = 360^o$
$\Rightarrow y = 20^o$
$\therefore\ \angle A = 40^o; \angle B = 80^o; \angle C = 100^o; \angle D = 140^o$

Answer 3:



Given , AB || DC. As we know that the interior angles on the same side of transversal line, then $\angle A = 55° \; and \; \angle B = 70°$

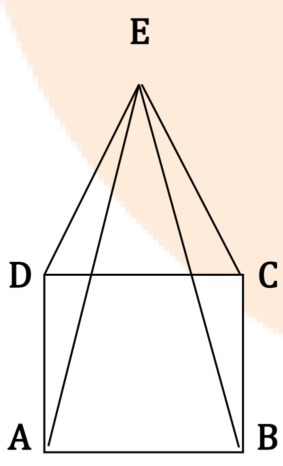
$\angle A + \angle D = 180°$

$\Rightarrow \angle D = 180° - \angle A = 180° - 55° = 125°$

Also , $\angle B + \angle C = 180°$

$\Rightarrow \angle C = 180° - \angle B = 180° - 70° = 110°$

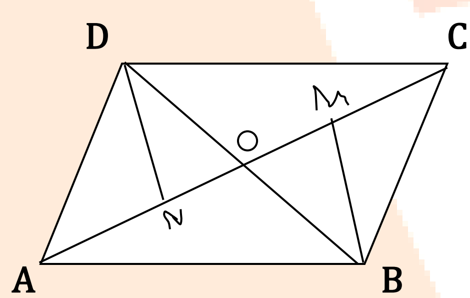
Answer 4:



Given: ABCD is a square in which AB = BC = CD = DA. ΔEDC is an equilateral triangle in which ED = EC = DC and ∠EDC = ∠DEC = ∠DCE = 60^o.
To prove: AE = BE and ∠DAE = 15^o
Proof: In ΔADE and ΔBCE, as ,
AD = BC [Sides of a square]

DE = EC [Sides of an equilateral triangle]
∠ADE = ∠BCE = $90^o + 60^o = 150^o$
∴ ΔADE ≅ ΔBCE
i.e., AE = BE
Now, ∠ADE = 150^o
DA = DC [Sides of a square]
DC = DE [Sides of an equilateral triangle]
So, DA = DE
ΔADE and ΔBCE are isosceles triangles.
i.e., $\angle DAE = \angle DEA = \frac{1}{2}(180° - 150°) = \frac{30}{2} = 15°$

Answer 5:



Given: by fig , both the diagonals intersect at O and BM ⊥ AC then
Let the diagonals intersect each other at O
Now, in ΔOND and ΔOMB,
∠OND = ∠OMB (90^o each)
∠DON = ∠BOM (Vertically opposite angles)

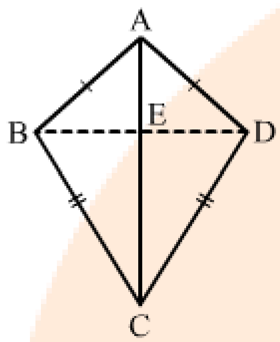
Also, DN = BM (Given)
As we know that by parallelogram

$\Delta OND \cong \Delta OMB$

$\therefore OD = OB$ HENCE PROVED

Hence, AC bisects BD.

Answer 6:



Given: ABCD is a quadrilateral in which AB = AD and BC = DC

(i) To prove : AC bisects ∠A and ∠C

In ΔABC and ΔADC,

AB = AD

BC = DC

AC is common in both the traiangles.

i.e., ΔABC ≅ ΔADC (SSS congruence rule)

∴ ∠BAC = ∠DAC and ∠BCA = ∠DCA (By CPCT)

Hence proved, AC bisects both the angles, ∠A and ∠ C.

(ii) To prove BE = DE

In ΔABE and ΔADE,

AB = AD

S∠BAE = ∠DAE

AE is common.

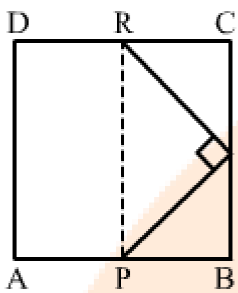
∴ ΔABE ≅ ΔADE (SAS congruence rule)

⇒ hence proved BE = DE

(iii) To prove : ∠ABC = ∠ADC

$\Delta ABC \cong \Delta ADC$ (Given)
Hence proved, $\angle ABC = \angle ADC$

Answer 7:



Given: ABCD is a square and $\angle PQR = 90°$.
$PB = QC = DR$
(i) To prove : $QB = DR$
$\therefore BC = CD$ (Sides of square)
and $CQ = DR$ (Given)
so , by fig $BC = BQ + CQ$
$\Rightarrow CQ = BC - BQ$
$\therefore DR = BC - BQ$ ...(i)
Also, $CD = RC + DR$
$\therefore DR = CD - RC = BC - RC$ ...(ii)
From (i) and (ii), we get
$BC - BQ = BC - RC$
$\therefore BQ = RC$

(ii)To prove, $PQ = QR$

In ΔRCQ and ΔQBP,
$PB = QC$ (Given)
$BQ = RC$ (Given)
$\angle RCQ = \angle QBP$ (90^o each)

By parallelogram theorem
$\Delta RCQ \cong \Delta QBP$ (SAS congruence rule)
$\therefore QR = PQ$ hence proved

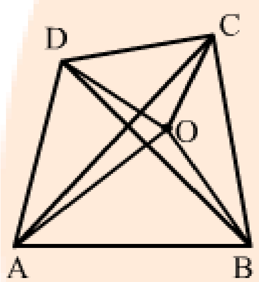
(iii) To prove, $\angle QPR = 45°$

$\Delta RCQ \cong \Delta QBP$ and $QR = PQ$

$\therefore$ In ΔRPQ, $\angle QPR = \angle QRP = \frac{1}{2}(180° - 90°) = \frac{90}{2} = 45°$

Hence proved, $\angle QPR = 45°$

Answer 8:

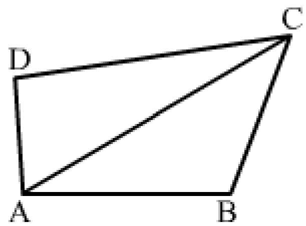


Let ABCD be a quadrilateral with diagonals AC and BD and O is a point within the quadrilateral.

Suppose
In ΔAOC, $OA + OC > AC$ ………….(1)

And, in ΔBOD, $OB + OD > BD$ ………………(2)
Adding these ,
$(OA + OC) + (OB + OD) > (AC + BD)$
$\Rightarrow OA + OB + OC + OD > AC + BD$

Answer 9:



Given: ABCD is a quadrilateral and AC is its diagonal.

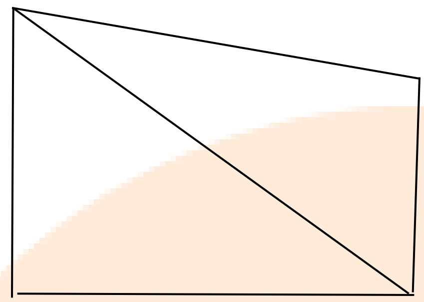
(i) As sum of any two sides of any triangle is greater than the third side.
In ΔABC, $AB + BC > AC$ ...(1)

In ΔACD, $CD + DA > AC$ ...(2)
Adding (1) and (2),
$AB + BC + CD + DA > 2AC$ ……………….hence proved

(ii) In ΔABC,
$AB + BC > AC$ ...(1)
In ΔACD,
$AC > |DA - CD|$ ...(2)
From (1) and (2),
$AB + BC > |DA - CD|$
$\Rightarrow AB + BC + CD > DA$…………………………hence proved

(iii) In ΔABC ,we know that $AB + BC > AC$
Same as, In ΔACD, $CD + DA > AC$
And
In Δ BCD,
$BC + CD > BD$
In Δ ABD,
$DA + AB > BD$
Adding these ,
$2(AB + BC + CD + DA) > 2(AC + BD)$
$\Rightarrow (AB + BC + CD + DA) > (AC + BD)$

Answer 10:

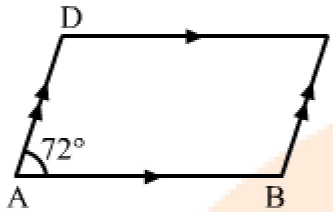


Let PQRS be a quadrilateral and $\angle 1, \angle 2, \angle 3$ and $\angle 4$ are its four angles .
Join QR which divides PQRS in two triangles, ΔPQR and ΔQRS.
In ΔPQR,
$\angle 1 + \angle 2 + \angle P = 180°$ ...(i)
In ΔQRS,
$\angle 3 + \angle 4 + \angle R = 180°$ ...(ii)
On adding (i) and (ii),

$(\angle 1 + \angle 3) + \angle P + \angle R + (\angle 4 + \angle 2) = 360°$
$\Rightarrow \angle P + \angle R + \angle Q + \angle S = 360°$ $\therefore \angle 1 + \angle 3 = \angle Q$; $\angle 4 + \angle 2 = \angle S$
Hence proved
$\therefore \angle P + \angle R + \angle Q + \angle S = 360°$

EXERCISE 10B

Answer 1:

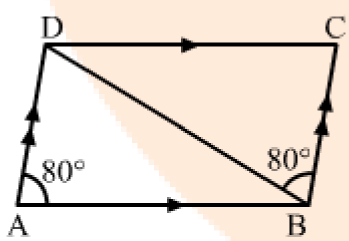


Given, ABCD is parallelogram and $\angle A = 72°$.

Then , as we know that opposite angles are equals.
$\therefore \angle A = \angle C$ and $\angle B = \angle D$
$\therefore \angle C = 72°$
$\angle A$ and $\angle B$ are the adjacent angles.
as, $\angle A + \angle B = 180°$
$\Rightarrow \angle B = 180° - \angle A = 180° - 72° = 108°$

As above, $\angle B = \angle D = 108°$
Hence, $\angle B = \angle D = 108^{o}$ and $\angle C = 72^{o}$

Answer 2:



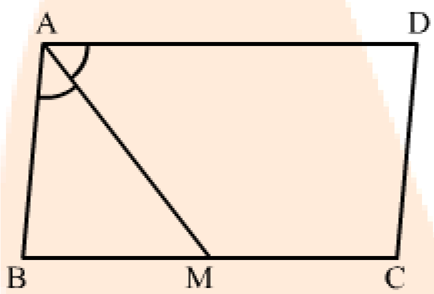
Given: ABCD is parallelogram and $\angle DAB = 80°$ *and* $\angle DBC = 60°$
To find: Measure of $\angle CDB$ and $\angle ADB$
In parallelogram ABCD, AD || BC
$\therefore \angle DBC = \angle ADB = 60°$ (Alternate interior angles) ...(i)
As $\angle DAB$ and $\angle ADC$ are the adjacent angles,

$\angle DAB + \angle ADC = 180°$

$\therefore \angle ADC = 180° - \angle DAB = 180° - 80° = 100°$
Also, $\angle ADC = \angle ADB + \angle CDB$
$\therefore \angle ADC = 100°$

Then,
$\Rightarrow \angle ADB + \angle CDB = 100$ ...(ii)
From (i) and (ii),
$60° + \angle CDB = 100°$
$\Rightarrow \angle CDB = 100° - 60° = 40$
Hence, $\angle CDB = 40^o$ and $\angle ADB = 60^o$

Answer 3:



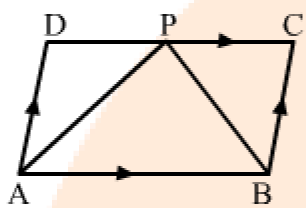
Given: parallelogram ABCD, M is the midpoint of side BC and
$\angle BAM = \angle DAM$.
To prove: AD = 2CD
Proof:
Since, AD∥BC and AM is the transversal.
So, $\angle DAM = \angle AMB$ (Alternate interior angles)
But, $\angle DAM = \angle BAM$ (Given)
Thus, $\angle AMB = \angle BAM$
$\Rightarrow AB = BM$

As we know angles opposite to equals sides are equal and opposite sides of parallelogram are equal
Now, $AB = CD$
$\Rightarrow 2AB = 2CD$

So, ⇒ $(AB + AB) = 2CD$
⇒$BM + MC = 2CD$ (AB = BM and MC = BM)
⇒$BC = 2CD$

∴ $AD = 2CD$ (AD=BC)hence proved

Answer 4:



ABCD is a parallelogram.
∴ ∠A = ∠C and ∠B = ∠D (Opposite angles)
And ∠A + ∠B = 180° (Adjacent angles are supplementary)
∴ ∠B = 180° − ∠A
⇒ 180° − 60° = 120° (∠A = 60°)
∴ ∠A = ∠C = 60° and ∠B = ∠D = 120°

(i) In Δ APB, ∠PAB =$\frac{60}{2}$ = 30°

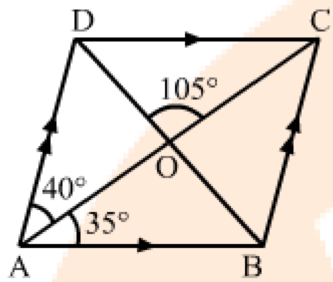
and ∠PBA = $\frac{120}{2}$ = 60°
∴ ∠APB = 180° − (30° + 60°) = 90°

(ii) In Δ ADP, ∠PAD = 30° and ∠ADP = 120°
∴ ∠APB = 180° − (30° + 120°) = 30°
Thus, ∠PAD = ∠APB = 30°
Hence, ΔADP is an isosceles triangle and AD = DP.
In Δ PBC, ∠PBC= 60°, ∠BPC= 180° −(90° +30°) =60° and∠BCP =60°
(Opposite angle of ∠A)
∴ ∠ PBC = ∠ BPC = ∠ BCP
Hence, ΔPBC is an equilateral triangle and, therefore, PB = PC = BC.

(iii) DC = DP + PC
From (ii), as ,

DC = AD + BC [AD = BC, opposite sides of a parallelogram]
⇒ DC = AD + AD
⇒ DC = 2 AD

Answer 5:



ABCD is a parallelogram.
∴ AB | | DC and BC | | AD
(i) In ΔAOB, $\angle BAO = 35°$,

As we know that, vertically opposite angles are equals

$\angle AOB = \angle COD = 105°$
$\therefore \angle ABO = 180° - (35° + 105°) = 40°$

(ii) As we know that these angles are ∠ODC and ∠ABO are alternate interior angles.
$\therefore \angle ODC = \angle ABO = 40°$

(iii) These are Alternate interior angles

$\angle ACB = \angle CAD = 40°$
(iv) In ΔABC, we get

$\angle CBD = \angle ABC - \angle ABD$ ...(i)

$\angle ABC = 180° - \angle BAD$ (Adjacent angles are supplementary)

$\Rightarrow \angle ABC = 180° - 75° = 105°$
In ΔCBD , we have
Then, ∠CBD = ∠ABC - ∠ABD

$\Rightarrow \angle CBD = 105° - \angle ABD$ (∠ABD = ∠ABO)
$\Rightarrow \angle CBD = 105° - 40° = 65°$

Answer 6:

ABCD is a parallelogram.
i.e., ∠A = ∠C and ∠B = ∠D (Opposite angles)
Also, $\angle A + \angle B = 180°$ (Adjacent angles are supplementary)
$\therefore (2x + 25)° + (3x - 5)° = 180°$
$\Rightarrow 5x + 20 = 180°$

$\Rightarrow 5x = 180 - 20$

$\Rightarrow 5x = 160°$
$\Rightarrow x = \frac{160}{2} = 32°$

$\therefore \angle A = 2 \times 32 + 25 = 89°$ and $\angle B = 3 \times 32 - 5 = 91°$
Hence, x = 32°, ∠A = ∠C = 89° and ∠B = ∠D = 91°

Answer 7:

Let PQRS be a parallelogram.
∴ ∠P = ∠R and ∠Q = ∠S
Let ∠P = y° and ∠B = $(\frac{4y}{5})°$
Now, ∠P + ∠Q = 180°
⇒y + $(\frac{4y}{5})°$ = 180°⇒$(\frac{9y}{5})°$ = 180°⇒y = 100°

Now, ∠P = 100° and ∠B = $(\frac{4}{5})\times 100°$ = 80°
Hence, ∠P = ∠R = 100°; ∠B = ∠S = 80°

Answer 8:

Let PQRS be a parallelogram.

$\therefore \angle P = \angle R$ and $\angle Q = \angle S$ (Opposite angles)

Let $\angle P$ be the smallest angle whose measure is y^o.

$\therefore \angle Q = (2y - 30)°$

$Now, \angle P + \angle Q = 180°$ (Adjacent angles are supplementry)

$\Rightarrow y + 2y - 30° = 180°$

$\Rightarrow 3y = 210°$

$\Rightarrow y = \frac{210}{3} = 70$

$\Rightarrow y = 70°$

$\therefore \angle Q = 2 \times 70° - 30° = 110°$

Hence, $\angle P = \angle R = 70^o$; $\angle Q = \angle S = 110^o$

Answer 9:

ABCD is a parallelogram.

The opposite sides of a parallelogram are parallel and equal.

$\therefore$ AB = DC = 9.5 cm

Let BC = AD = y

$\therefore$ Perimeter of ABCD = $AB + BC + CD + DA = 30\ cm$

$\Rightarrow 9.5 + y + 9.5 + y = 30$

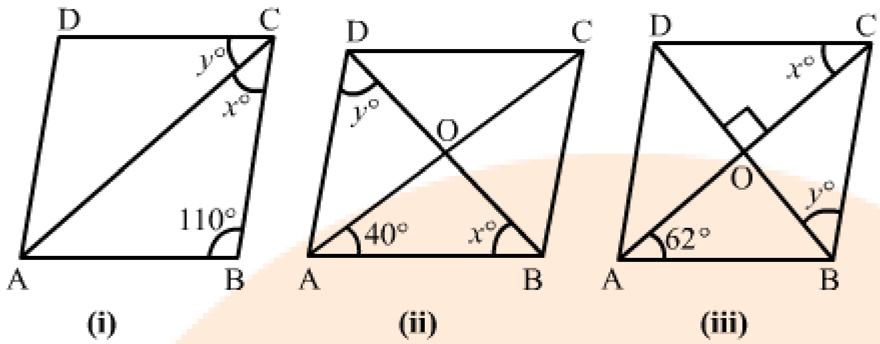
$\Rightarrow 19 + 2y = 30$

$\Rightarrow 2y = 11$

$\Rightarrow y = \frac{11}{2} = 5.5\ cm$

Hence, AB = DC = 9.5 cm and BC = DA = 5.5 cm

Answer 10:



ABCD is a rhombus and a rhombus is also a parallelogram. A rhombus has four equal sides.

(i) In ΔABC,

$\angle BAC = \angle BCA = \frac{1}{2}(180 - 110)° = 35°$

i.e., x = 35^o

Now by Adjacent angles are supplementary we get,

$\angle B + \angle C = 180°$

As, $\angle C = x + y = 70°$

$\Rightarrow y = 70° - x$

$\Rightarrow y = 70° - 35° = 35°$

Hence, x = 35^o; y = 35^o

(ii) The diagonals of a rhombus are perpendicular bisectors of each other.

So, in ΔAOB, ∠OAB = 40^o, ∠AOB = 90^o and

∠ABO + ∠BOA +∠ OAB = 180

$\angle ABO = 180° - (40° + 90°) = 50°$

∴ x = 50^o

In ΔABD, AB = AD

So, $\angle ABD = \angle ADB = 50°$

Hence, x = 50^o; y = 50^o

(iii) ∠BAC = ∠DCA (Alternate interior angles)

i.e., x = 62°

In ΔBOC, ∠BCO = 62°

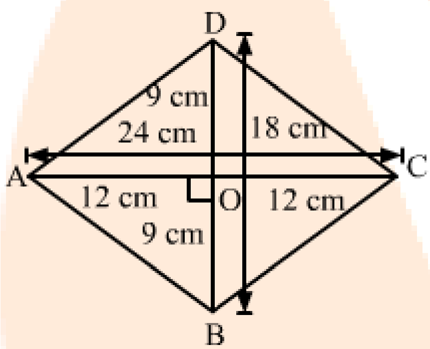
Also, ∠BOC = 90°

∠BCO +∠BOC +∠OBC = 180

$\therefore \angle OBC = 180° - (90° + 62°) = 28°$

Hence, x = 62°; y = 28°

Answer 11:



Let PQRS be a rhombus.

∴ PQ = QR = RS = SP

Here, PR and QS are the diagonals of PQRS, where PR = 24 cm and QS = 18 cm.

Let the diagonals intersect each other at M.

∴ ΔPMQ is a right angle triangle in which $MP = \frac{AC}{2} = \frac{24}{2} = 12$ cm and $MQ = \frac{QS}{2} = \frac{18}{2} = 9$ cm.

Now, $PQ^2 = MP^2 + MQ^2$ [Pythagoras theorem]

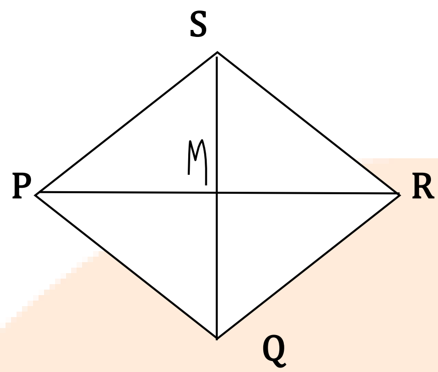
$\Rightarrow PQ^2 = (12)^2 + (9)^2$

$\Rightarrow PQ^2 = 144 + 81 = 225$

⇒ PQ= 15 cm

Hence, the side of the rhombus is 15 cm.

Answer 12:



Let PQRS be a rhombus.

∴ PQ = QR = RS = SP = 10 cm

Let PR and QS are the diagonals of PQRS. Let PR = y and QS = 16 cm and M be the intersection point of the diagonals.

∴ ΔPMQ is a right angle triangle, in which

MP $= \frac{PR}{2} = \frac{y}{2}$ and MQ $= \frac{QS}{2} = \frac{16}{2} = 8$ cm.

Now, $PQ^2 = MP^2 + MQ^2$ [Pythagoras theorem]

$\Rightarrow 10^2 = (\frac{y}{2})^2 + 8^2 \Rightarrow 100 - 64 = \frac{y2}{4} \Rightarrow 36 \times 4 = y^2$

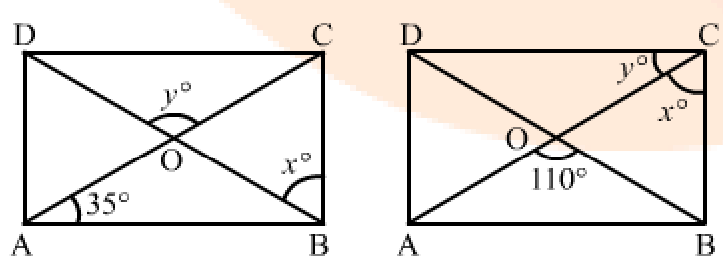
$\Rightarrow y^2 = 144$

∴ y = 12 cm

Hence, the other diagonal of the rhombus is 12 cm.

∴ Area of the rhombus = 12×(12×16) = 96 cm^2

Answer 13:



(i) (ii)

(i) ABCD is a rectangle.
The diagonals of a rectangle are congruent and bisect each other. Therefore, in Δ AOB, as ,

$OA = OB$

$\therefore \angle OAB = \angle OBA = 35^o$

$\therefore x = 90^o - 35^o = 55^o$

In ΔAOB

$\angle OAB + \angle OBA + \angle AOB = 180^\circ$

And $\angle AOB = 180^o - (35^o + 35^o) = 110^o$

$\therefore y = \angle AOB = 110^o$ [Vertically opposite angles]

Hence, $x = 55^o$ and $y = 110^o$

(ii) In ΔAOB, as ,

Given, $\angle AOB = 100\,^\circ$

$OA = OB$

As, $\angle OAB = \angle OBA$

Then, $\angle AOB + \angle OBA + \angle OAB = 180$

$\Rightarrow 2\angle AOB = 180 - \angle AOB$ ....($\angle OAB = \angle OBA$)

$\Rightarrow 2\angle AOB = 180 - 110 = 70^\circ$

$\Rightarrow \angle AOB = \frac{1}{2} \times 70 = 35^\circ$

so, $\therefore y = \angle BAC = 35^o$ [Interior alternate angles]

Here at $\angle C$ is at right angle Δ by fig,

$\Rightarrow 90^o = x + y$

$\Rightarrow x = 90^o - y$

$\Rightarrow x = 90^o - 35^o = 55^o$

Thus, $x = 55^o$ and $y = 35^o$

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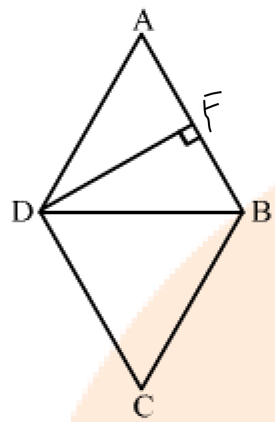
Answer 14:



Given: ABCD is a rhombus, DF is altitude which bisects AB i.e. AF = FB

In ΔAFD and ΔBFD,

DF=DF (Common side)

∠DFA=∠DFB=90° (Given)

AF=FB (Given)

∴ ΔAFD≅ΔBFD (By SAS congruence Criteria)

⇒AD=BD (CPCT)

Also, AD=AB (Sides of rhombus are equal)

⇒AD=AB=BD

Thus, ΔABD is an equilateral triangle.

Therefore, ∠A=60°

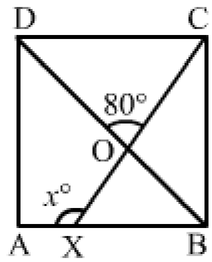
⇒∠C=∠A=60° (Opposite angles of rhombus are equal)

And, ∠ABC+∠BCD=180° (Adjacent angles of rhombus are supplementary.)

⇒∠ABC+60°=180°⇒∠ABC=180°−60°⇒∠ABC=120°⇒∠ADC=∠ABC=120°

Hence, the angles of rhombus are 60°, 120°, 60° and 120°

Answer 15:

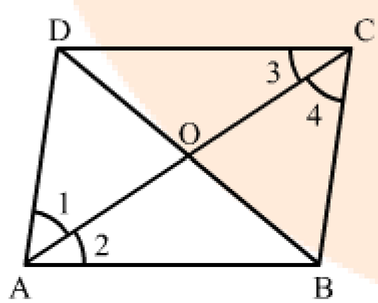


The angles of a square are bisected by the diagonals.

$\angle OBX = \frac{1}{2} \times \angle CBA = \frac{1}{2} \times 90 = 45°$
$\therefore \angle OBX = 45°$
Given, $\angle COD = 80°$

And $\angle BOX = \angle COD = 80°$ [Vertically opposite angles]
$\therefore$ In ΔBOX, as we know that exterior angle is sum of both interior angles.
$\angle AXO = \angle OBX + \angle BOX$
$\Rightarrow \angle AXO = 45° + 80° = 125°$
$\therefore x = 125°$

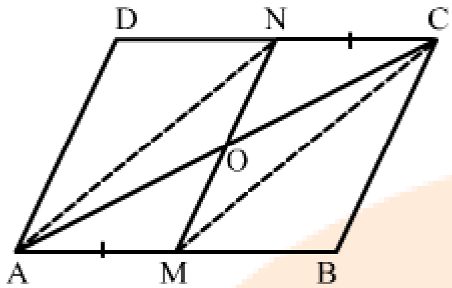
Answer 16:



Given: A rhombus ABCD.
To prove: Diagonal AC bisects ∠A as well as ∠C and diagonal BD bisects ∠B as well as ∠D.
Proof:

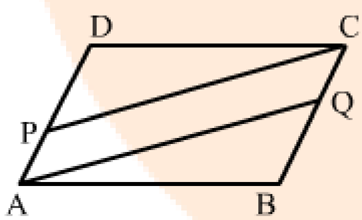
In ΔABC,
$AB = BC$ (Sides of rhombus are equal.)
$\angle ACB = \angle CAB$ (Angles opposite to equal sides are equal.) ...(1)
AD∥BC (Opposite sides of rhombus are parallel.)
AC is transversal.
$\angle DAC = \angle ACB$ (Alternate interior angles) ...(2)
From (1) and (2),
$\angle DAC = \angle CAB$
Thus, AC bisects ∠A.
As, AB∥DC and AC is transversal.
$\angle CAB = \angle DCA$ (Alternate interior angles) ...(3)
From (1) and (3),
$\angle ACB = \angle DCA$
Thus, AC bisects ∠C.
Thus, AC bisects ∠C and ∠A
In ΔDAB,
$AD = AB$ (Sides of rhombus are equal.)
$\angle ADB = \angle ABD$ (Angles opposite to equal sides are equal.) ...(4)
Also,
DC∥AB (Opposite sides of rhombus are parallel.)
BD is transversal.
$\angle CDB = \angle DBA$ (Alternate interior angles) ...(5)
From (4) and (5),
$\angle ADB = \angle CDB$
Therefore, DB bisects ∠D.
As, AD∥BC and BD is transversal.
∠CBD=∠ADB (Alternate interior angles) ...(6)
From (4) and (6)
$\angle CBD = \angle ABD$
Therefore, BD bisects ∠B.
Thus, BD bisects ∠D and ∠B.

Answer 17:



Given: In a parallelogram ABCD, AM = CN.
To prove: AC and MN bisect each other.
Construction: Join AN and MC.
Proof:
As, ABCD is a parallelogram.
$\Rightarrow AB \parallel DC \Rightarrow AM \parallel NC$
And, $AM = CN$ (Given)
Therefore, AMCN is a parallelogram.
As, the diagonals of a parallelogram bisect each other.
Thus, AC and MN also bisect each other.

Answer 18:



As , per by given fig,
$\angle B = \angle D$ [Opposite angles of parallelogram ABCD]
$AD = BC \text{ and } AB = DC$ [Opposite sides of parallelogram ABCD]
Also, AD || BC and AB|| DC

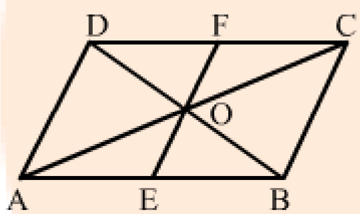
Given, AP $= \frac{1}{3}$AD and CQ $= \frac{1}{3}$BC

So, we get
$\therefore AP = CQ$ [AD = BC]

In ΔDPC and ΔBQA,
$AB = CD, \angle B = \angle D \text{ and } DP = QB$ [DP $= \frac{2}{3}$AD and QB $= \frac{2}{3}$BC]
i.e., $\Delta DPC \cong \Delta BQA$
$\therefore PC = QA$

Thus, in quadrilateral AQCP,
$AP = CQ$ ...(i)
$PC = QA$ ...(ii)
∴ AQCP is a parallelogram.

Answer 19:

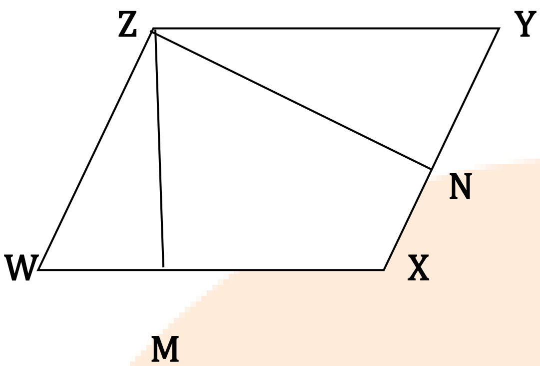


Given, ABCD is a parallelogram whose diagonals intersect each other at O. A line segment EOF is drawn to meet AB at E and DC at F.

So in ΔODF and ΔOBE,
$OD = OB$ (Diagonals bisects each other)
$\angle DOF = \angle BOE$ (Vertically opposite angles)
$\angle FDO = \angle OBE$ (Alternate interior angles)

By parallelogram theorem
$\Delta ODF \cong \Delta OBE$
$\therefore OF = OE$
Hence, proved.

Answer 20:



Given: parallelogram WXYZ, ZM⊥ WX, WN ⊥ XY and ∠MZN = 60°

In quadrilateral ZMXN, by angle sum property,

$$\angle MZN + \angle ZMX + \angle X + \angle XNZ = 360°$$

$\Rightarrow 60° + 90° + \angle X + 90° = 360°$

$\Rightarrow \angle X = 360° - 240° \Rightarrow \angle X = 120° \Rightarrow \angle X = 120°$

Also,

$\angle X = \angle Z = 120°$ (Opposite angles of a parallelogram are equal.)

$\angle W + \angle X = 180°$ (Adjacent angles of a parallelogram are supplementary.)

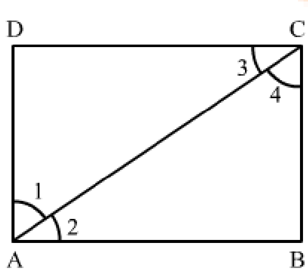
$\Rightarrow \angle W + 120° = 180° \Rightarrow \angle W = 180° - 120° \Rightarrow \angle W = 60°$

Also,

$\angle W = \angle Y = 60°$ (Opposite angles of a parallelogram are equal.)

Thus, the angles of a parallelogram are 60°, 120°, 60° and 120°.

Answer 21:

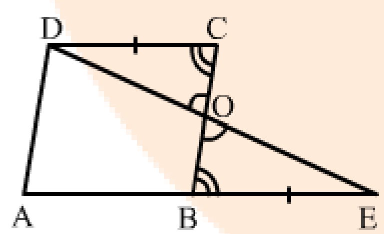


Given: In
rectangle ABCD, AC bisects ∠A, i.e. ∠DAC = ∠CAB and AC bisects ∠C, i.e. ∠D
CA = ∠ACB.
To prove:
(i) ABCD is a square,
(ii) diagonal BD bisects ∠B as well as ∠D.
Proof:
(i) Since, AD∥BC (Opposite sides of a rectangle are parallel.)
So, ∠DAC=∠ACB (Alternate interior angles)
But, ∠DAC=∠CAB (Given)

So, ∠CAB = ∠ACB
In ΔABC,
Since, ∠CAB=∠ACB
So, BC=AB (Sides opposite to equal angles are equal.)
But these are adjacent sides of the rectangle ABCD.
Hence, ABCD is a square.

(ii) Since, the diagonals of a square bisects its angles.
So, diagonals BD bisects ∠B as well as ∠D.

Answer 22:



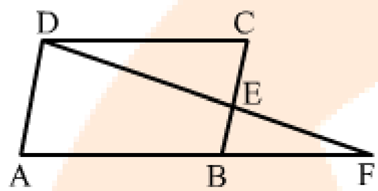
Given, ABCD is parallelogram in which AB is produced to E.

BE = AB (given)

So in ΔODC and ΔOEB, as ,
$DC = BE$ (DC = AB)

$\angle OCD = \angle OBE$ (Alternate interior angles)
$\angle COD = \angle BOE$ (Vertically opposite angles)
by parallelogram theorem we get,
$\therefore \Delta ODC \cong \Delta OEB$
$\Rightarrow OC = OB$
Hence , ED bisects BC.

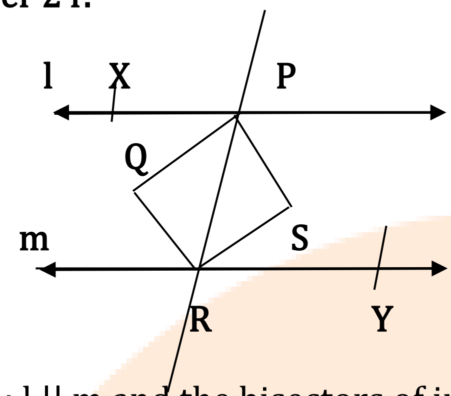
Answer 23:



Given: ABCD is a parallelogram.
BE = CE
DE and AB when produced meet at F.
To prove: AF = 2AB
Proof: In parallelogram ABCD, as ,
$AB \parallel DC$
$\angle DCE = \angle EBF$ (Alternate interior angles)
In ΔDCE and ΔBFE,
$\angle DCE = \angle EBF$ (Proved above)

$\angle DEC = \angle BEF$ (Vertically opposite angles)
And, $BE = CE$ (Given)
By parallelogram theorem
$\therefore \Delta DCE \cong \Delta BFE$
hence $\therefore DC = BF$
But DC = AB, as ABCD is a parallelogram.
$\therefore DC = AB = BF$ ...(i)
can also be written as , $AF = AB + BF$ ...(ii)
$AF = AB + AB = 2AB$ ………...from(i)
Hence, proved. AF = 2AB.

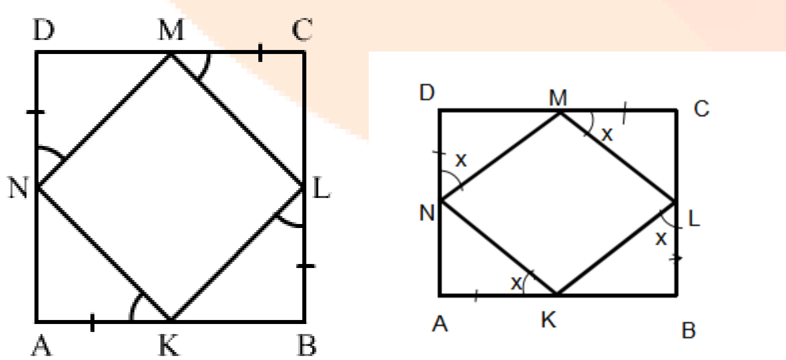
Answer 24:



Given: l || m and the bisectors of interior angles intersect at X and Y.
To prove: PQRS is a rectangle.
Proof:
Since, l || m (Given)
So, $\angle XPR = \angle PRY$ (Alternate interior angles)
$\Rightarrow \frac{1}{2}\angle XPR = \frac{1}{2}\angle PRY$
$\Rightarrow \angle QPR = \angle PRS$ but, these are a pair of alternate interior angles for PQ and RS.
$\Rightarrow PQ \parallel SR$
Similarly, $PR \parallel QS$
So, PQRS is a parallelogram.
Also,`
$\angle XPR + \angle RPZ = 180°$ (Linear pair)
$\Rightarrow \frac{1}{2}\angle XPR + \frac{1}{2}\angle PRY = 90° \Rightarrow \angle QPR + \angle RPS = 90° \Rightarrow \angle QPS = 90°$

But, this an angle of the parallelogram PQRS
Hence, PQRS is a rectangle.

Answer 25:

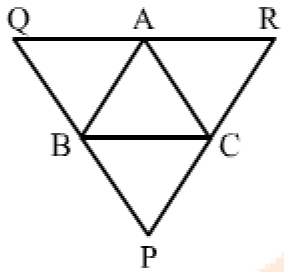


Given: In square ABCD, AK = BL = CM = DN.
To prove: KLMN is a square.
Proof:
In square ABCD,
$AB = BC = CD = DA$ (All sides of a square are equal.)
And, $AK = BL = CM = DN$ (Given)
So, $AB - AK = BC - BL = CD - CM = DA - DN$
$\Rightarrow KB = CL = DM = AN$ ...(1)
In ΔNAK and ΔKBL,
$\angle NAK = \angle KBL = 90°$ (Each angle of a square is a right angle.)
$AK = BL$ (Given)
$AN = KB$ [From (1)]
So, by parallelogram theorem ,
$\Delta NAK \cong \Delta KBL$
$\Rightarrow NK = KL$ (CPCT) ...(2)
Similarly,
$\Delta MDN \cong \Delta NAK \ \Delta DNM \cong CML \Delta MCL \cong LBK$
$\Rightarrow MN = NK \ and \ \angle DNM = \angle KNA$ (CPCT) ...(3)
$MN = JM \ and \ \angle DNM = \angle CML$ (CPCT) ...(4)
$ML = LK \ and \ \angle CML = \angle BLK$ (CPCT) ...(5)

From (2), (3), (4) and (5),
$NK = KL = MN = ML$ ...(6)
And, $\angle DNM = \angle AKN = \angle KLB = LMC$
Now,
In ΔNAK,
$\angle NAK = 90°$
Let ∠AKN = y°
So, $\angle DNK = 90° + y°$
$\Rightarrow \angle DNM + \angle MNK = 90° + y° \Rightarrow y° + \angle MNK = 90° + y° \Rightarrow \angle MNK = 90°$
Similarly,
$\angle NKL = \angle KLM = \angle LMN = 90°$ ...(7)
Using (6) and (7),
All sides of quadrilateral KLMN are equal and all angles are 90°

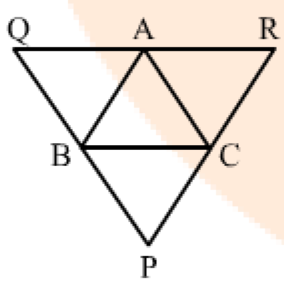
So, KLMN is a square.

Answer 26:



Δ ABC , if lines are drawn through A, B, C parallel respectively to the sides BC, CA and AB. So, we get, BC || QA and CA || QB
i.e., BCQA is a parallelogram.
$\therefore BC = QA$ ...(i)
Similarly, BC || AR and AB || CR.
i.e., BCRA is a parallelogram.
$\therefore BC = AR$ ...(ii)
As $QR = QA + AR$
From (i) and (ii),
$QR = BC + BC$
$\Rightarrow QR = 2BC$
$\therefore$ BC $= \frac{1}{2}$QR

Answer 27:



In ΔABC A, B, C lines drawn, parallel respectively to BC, CA and AB intersecting at P , Q and R. Acc to question,

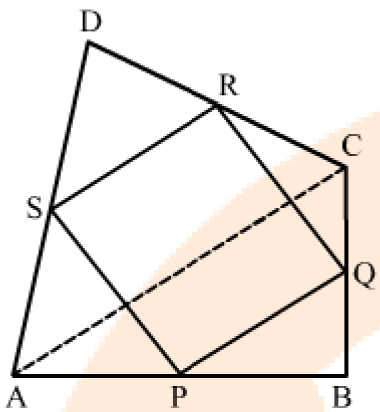
$Perimeter\ of\ \Delta ABC = AB + BC + CA$ ...(i)
$Perimeter\ of\ \Delta PQR = PQ + QR + PR$ ...(ii)
By given figure,

BC || QA and CA || QB
i.e., BCQA is a parallelogram.
$\therefore BC = QA$ ...(iii)
Similarly, BC || AR and AB || CR
i.e., BCRA is a parallelogram.
$\therefore BC = AR$ ...(iv)
But, $QR = QA + AR$
From (iii) and (iv),
$\Rightarrow QR = BC + BC$
$\Rightarrow QR = 2BC$
$\therefore \text{BC} = \frac{1}{2}\text{QR}$
Similarly, $\text{CA} = \frac{1}{2}\text{PQ}$ and $\text{AB} = \frac{1}{2}\text{PR}$

From (i) and (ii),
Perimeter of $\Delta\text{ABC} = \frac{1}{2}\text{QR} + \frac{1}{2}\text{PQ} + \frac{1}{2}\text{PR}$
$= \frac{1}{2}(\text{PR} + \text{QR} + \text{PQ})$
i.e., Perimeter of $\Delta\text{ABC} = \frac{1}{2}(\text{Perimeter of }\Delta\text{PQR})$
$\therefore \textit{Perimeter of } \Delta PQR = 2 \times \textit{Perimeter of } \Delta ABC$

EXERCISE – 10C

Answer 1:



Given: In
quadrilateral ABCD, P, Q, R and S are respectively the midpoints of the sides A
B, BC, CD and DA.
To prove:
(i) PQ || AC and PQ = $\frac{1}{2}$AC
(ii) PQ || SR
(iii) PQRS is a parallelogram.
Proof:
(i) In ΔABC,
Since, P and Q are the mid points of sides AB and BC, respectively. (Given)
⇒AC∥PQ and PQ=$\frac{1}{2}$AC (Using mid-point theorem.)
(ii) In ΔADC,
Since, S and R are the mid-points of AD and DC, respectively. (Given)
⇒SR∥AC and SR=$\frac{1}{2}$AC (Using mid-point theorem.) ...(1)
From (i) and (1), we get
PQ || SR
(iii) From (i) and (ii), we get
PQ=SR=$\frac{1}{2}$AC
So, PQ and SR are parallel and equal.
Hence, PQRS is a parallelogram.

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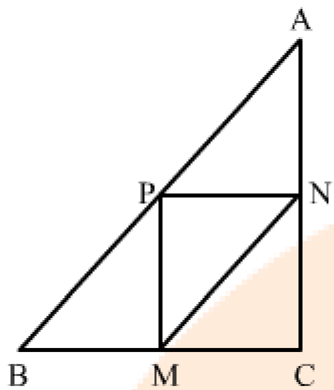
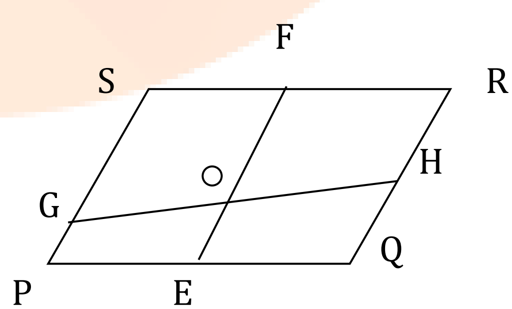
Answer 2:



Given: In an isosceles right ΔXYZ, ZEFG is a square.
To prove: F bisects the hypotenuse XY i.e., XF = FY.
Proof:
In square ZEFG,
∴ ZE = EF = FG = ZG (All sides are equal.)
Also, ΔXYZ is an isosceles with XZ = YZ.
⇒ XG + GZ = ZE + EY
⇒ XG = EY (ZG = ZE) ...(i)
Now,
In ΔXGF and ΔFEY,
XG = EY [From (i)]
∠XGP = ∠FEY = 90°
FG = FE (Sides of square CEFG)
∴ By SAS congruence criteria,
ΔXGF ≅ ΔYEF
Hence, XF = FY (By CPCT)

Answer 3:

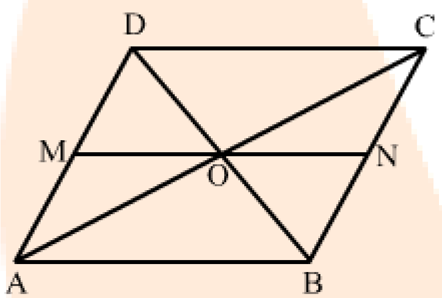
In parallelogram PQRS,
PS || QR and PQ || RS



$PS = QR \text{ and } PQ = SR$
$PQ = PE + QE \text{ and } RS = SF + FR$
$\therefore\ PE = QE = SF = FR$
Now, $SF = PE \text{ and } SF \parallel PE$.

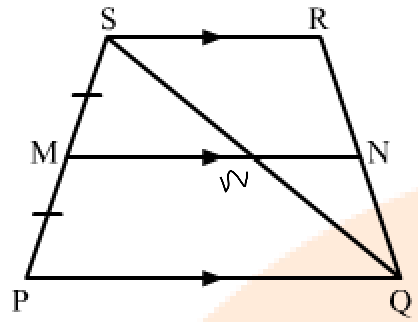
i.e., PEFS is a parallelogram.
∴ PS|| EF
Similarly, QEFR is also a parallelogram.
∴ EF || QR
∴ $PS \parallel EF \parallel QR$
Thus, PS, EF and QR are three parallel lines cut by the transversal line SR at S, F and R, such that SF = FR.
These lines PS, EF and QR are also cut by the transversal PQ at P, E and Q, such that PE = QE.
Similarly, they also cut by GH.
∴ GO = OH (By intercept theorem)

Answer 4:



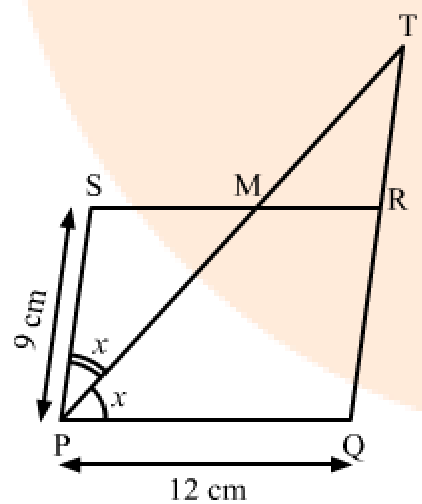
Given: A parallelogram ABCD
To prove: MN is bisected at O
Proof:
In ΔOAM and ΔOCN, we get by fig,
$OA = OC$ (Diagonals of parallelogram bisect each other)
$\angle AOM = \angle CON$ (Vertically opposite angles)
$\angle MAO = \angle OCN$ (Alternate interior angles)
∴ By ASA congruence criteria, and parallelogram theorem
$\Delta OAM \cong \Delta OCN$
$\Rightarrow OM = ON$
Hence proved , MN is bisected at O.

Answer 5:



Given: In trapezium PQRS, PQ || SR, M is the midpoint of PS and MN || PQ.
To prove: N is the midpoint of QR.
Construction: Join QS.
Proof:
In ΔSPQ, we get
M is the mid-point of SP and MW || PQ.
Therefore, W is the mid-point of SQ. (By Mid-point theorem)
Also, in ΔSRQ,
As, W is mid-point of SQ and WN || SR
Therefore, N is the mid-point of QR. (By Mid-point theorem)

Answer 6:



Given: In parallelogram PQRS, PQ = 12 cm and PS = 9 cm. The bisector of ∠SPQ meets SR at M.

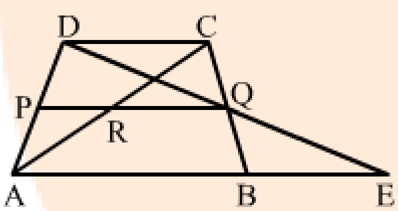
Let ∠SPQ = 2y.
⇒ ∠SRQ = 2y and ∠TPQ = y.
Also, PQ | | SR
⇒ ∠TMR = ∠TPQ = y.
In ∠TMR, ∠SRQ is an exterior angle.
⇒ ∠SRQ = ∠TMR + ∠MTR
⇒ 2y = y + ∠MTR
⇒ ∠MTR = y
⇒ ∠TPQ is an isosceles triangle.
⇒ TQ = PQ = 12 cm
Now,
RT = TQ – QR

= TQ − PS

= 12 − 9

= 3 cm

Answer 7:



Given: AB || DC, AP = PD and BQ = CQ

(i) In ΔQCD and ΔQBE,
$\angle DQC = \angle BQE$ (Vertically opposite angles)

$\angle DCQ = \angle EBQ$ (Alternate angles, as AE || DC)
$BQ = CQ$ (P is the midpoints)

By parallelogram theorem
$\therefore \Delta QCD \cong \Delta QBE$
hence $\therefore, DQ = QE$

(ii) Now, in ΔADE, P and Q are the midpoints of AD and DE, respectively.
∴ PQ || AE from above

From fig we get,

⇒ PQ || AB || DC

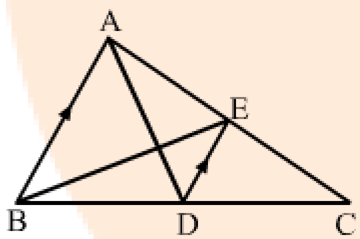
R is intersect point on AC and PQ then,
⇒ AB || PR || DC

(iii) PQ, AB and DC are the three lines cut by transversal AD at P such that

AP = PD.
These lines PQ, AB, DC are also cut by transversal BC at Q such that

BQ = QC.
Also, lines PQ, AB and DC are also cut by AC at R.
$\therefore\ AR = RC$

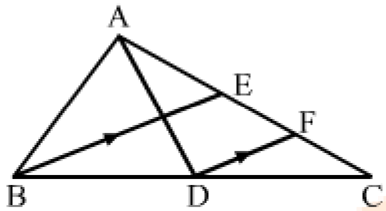
Answer 8:



AD is a median of ΔABC.

D is the mid point BC
$\therefore BD = DC$
It is clear that the line drawn through the midpoint of one side of triangle and parallel to another side bisects the third side.
Then DE bisects AC.
∴, AE = EC
∴ E is midpoint of AC.
⇒ BE is median in ΔABC.

Answer 9:



In ΔABC, by fig, we get
$AC = AE + EC$ ...(i)
E is point of AC, then

$AE = EC$

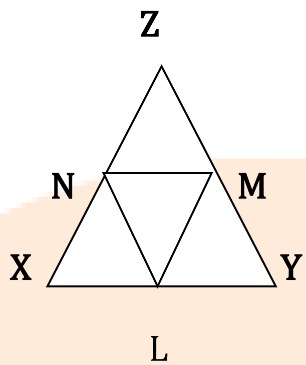
Can also be written as
$\therefore AC = 2EC$ ...(iii)

In ΔBEC, DF || BE.

F is mid point of EC
$\therefore EF = CF$
As, $EC = EF + CF$
$\Rightarrow EC = 2 \times CF$ ...(iv)
From (iii) and (iv),
$AC = 2 \times (2 \times CF)$

$AC = 4 \times CF$
$\therefore \text{ CF } = \frac{1}{4}\text{AC}$

Answer 10



ΔXYZ is given. L, M and N are the midpoints of sides XY, YZ and ZX, respectively.
As, L and M are the mid points of sides XY, and YZ of Δ XYZ.
∴ LM | | XZ (By midpoint theorem)
Similarly, LN | | YZ and MN | | XY.
Therefore, XLMN, YLNM and LNZM are all parallelograms.
Now, LM is the diagonal of the parallelogram YLNM.
∴ ΔYLM ≅ ΔNML
Similarly, LN is the diagonal of the parallelogram XLMN.

∴ ΔLXN ≅ ΔNML
And, MN is the diagonal of the parallelogram LNZM.
∴ ΔMNZ ≅ ΔNML
So, all the four triangles are congruent.

Answer 11:

D, E and F are the midpoints of sides BC, CA and AB, respectively.
As F and E are the mid points of sides AB and AC of Δ ABC.
∴ FE | | BC (By mid point theorem)
Similarly, DE | | FB and FD | | AC.
Therefore, AFDE, BDEF and DCEF are all parallelograms.
In parallelogram AFDE, as ,
$\angle A = \angle EDF$ (Opposite angles are equal)
In parallelogram BDEF, as ,

$\angle B = \angle DEF$ (Opposite angles are equal)
In parallelogram DCEF, as ,
$\angle C = \angle DFE$ (Opposite angles are equal)

Answer 12:

Let LMNO be the rectangle and E, F, G and H be the midpoints of LM, MN, NO and OL, respectively.
Join LN, a diagonal of the rectangle.
In Δ LMN, as ,
$\therefore EF \,||\, LN$ and $EF = \frac{1}{2}\,LN$ [By midpoint theorem]
Again, in Δ OLN, the points G and H are the mid points of LO and ON, respectively.
$\therefore GH \,||\, LN$ and $GH = \frac{1}{2}\,LN$ [By midpoint theorem]
Now, EF || LN and GH || LN
$\Rightarrow EF \,||\, GH$

Also, $EF = GH$ [Each equal to $\frac{1}{2}$ LN] ...(i)
So, EF GH is a parallelogram.
Now, in ΔHLE and ΔFME, as ,
$LH = MF$
$\angle L = \angle M = 90^o$
$LE = ME$
i.e., $\Delta HLE \cong \Delta FME$
$\therefore EH = EF$ ...(ii)
Similarly, $\Delta HOG \cong \Delta FNG$
$\therefore HG = GF$ ...(iii)
From (i), (ii) and (iii), as ,
$EF = EF = HG = HG$
Hence, EFGH is a rhombus.

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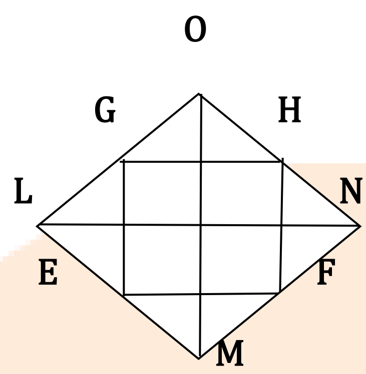
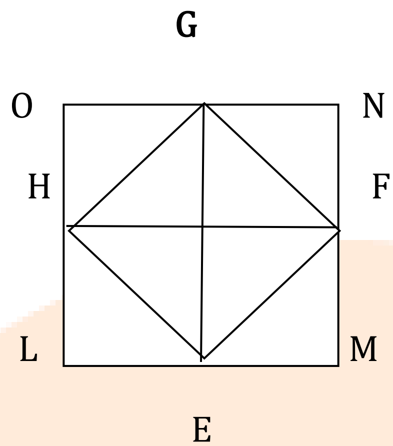
Answer 13:



Let LMNO be the rectangle and E, F, G and H be the midpoints of LM, MN, NO and OL.
Join the diagonals, LN and MO.
In Δ LMN,
∴ EF | | LN and EF $= \frac{1}{2}$ LN [By midpoint theorem]
Now, in Δ OLN, the points G and H are mid points of LO and ON .
∴ GH | | LN and GH $= \frac{1}{2}$ LN [By midpoint theorem]
As, $EF \mid\mid LN \text{ and } GH \mid\mid LN$
$\Rightarrow EF \mid\mid GH$

Also, EF = GH ...(i)
∴, EF GH is a parallelogram.
∴ $\angle YKX = 90°$
Now, $XG \mid\mid KM$
$\Rightarrow GY \mid\mid FK$
Also, $HG \mid\mid LN$
$\Rightarrow XG \mid\mid KY$
∴ KYGX is a parallelogram.
∴, $\angle XGY = \angle YKX = 90°$
Thus, EFGH is a parallelogram with ∠G = 90°.
∴ EFGH is a rectangle.

Answer 14:



Let LMNO be the rectangle and E, F, G and H be the midpoints of LM, MN, NO and OL, respectively.
Join the diagonals LN and MO. Let OM cut HG at X and LN cut FG at Y. Let K be the intersection point of LN and OM.
In Δ LMN, as ,

∴ EF | | LN and EF = $\frac{1}{2}$ LN [By midpoint theorem]

Again, in Δ OLN, the points G and H are the mid points of LO and ON respectively.

∴ GH | | LN and GH = $\frac{1}{2}$ LN [By midpoint theorem]

Now, EF | | LN and GH | | LN
⇒ EF | | GH

Also, EF = GH [Each equal to $\frac{1}{2}$ LN] ...(i)

So, EF GH is a parallelogram.
Now, in ΔHLE and ΔFME, as ,
LH = MF
∠L = ∠M = 90°
LE = ME
i.e., ΔHLE ≅ ΔFME
∴ EH = EF ...(ii)
Similarly, ΔSDR ≅ ΔRCQ
∴ HG = FG ...(iii)
From (i), (ii) and (iii), as ,
EF = EF = HG = HG...(iv)
We know that the diagonals of a square bisect each other at right angles.
∴ ∠XOY = 90°

Now, GQ | | ON
⇒GX | | YO
Also, HG | | LN
⇒YG | | KX
∴ KXRY is a parallelogram.
So, $\angle YRX = \angle XKY = 90^{o}$ (Opposite angles are equal)
Thus, EFGH is a parallelogram with $\angle G = 90^{o}$ and EF = EF = HG = HG.
∴ EFGH is a square.

Answer 15:

Let LMNO be the rectangle and E, F, G and H be the midpoints of LM, MN, NO and OL, respectively.
Join EF, FG, GH, HE and NO. NO is a diagonal of LMNO.
In Δ LMN, as ,
∴ EF | | LN and EF $= \frac{1}{2}$ LN (i) (By midpoint theorem)
Similarly in Δ MNO, as ,
∴ GH | | LN and GH $= \frac{1}{2}$ MO (ii) (By midpoint theorem)
From equations (i) and (ii), we get:
HE || MO || FG∴ HE || FG and HE = FG [Each equal to $\frac{1}{2}$ MO]
In quadrilateral HEFG, one pair of the opposite sides is equal and parallel to each other.
∴ HEFG is a parallelogram.
We know that the diagonals of a parallelogram bisect each other.
∴ EG and FH bisect each other.

Answer 16

Given: In quadrilateral ABCD, BD = AC and E, F, G and H are the mid-points of AD, CD, BC and AB, respectively.
To prove: EFGH is a rhombus.
Proof:
In ΔADC,

Since, E and F are the mid-points of sides AD and CD, respectively.

So, EF || AC and EF $= \frac{1}{2}$ AC ...(1)

Similarly, in ΔABC,

Since, G and H are the mid-points of sides BC and AB, respectively.

So, GH || AC and GH $= \frac{1}{2}$ AC ...(2)

From (1) and (2), we get

EF = GH and EF || GH

But this a pair of opposite sides of the quadrilateral EFGH.

So, EFGH is a parallelogram.

Now, in ΔABD,

Since, F and G are the mid-points of sides AD and AB, respectively.

So, FG || BD and FG $= \frac{1}{2}$ BD ...(3)

But BD = AC (Given)

$\Rightarrow \frac{1}{2}$ BD $= \frac{1}{2}$ AC

⇒FG = GH [From (2) and (3)]

But these are a pair of adjacent sides of the parallelogram EFGH.

Hence, EFGH is a rhombus.

Answer 17:

Given: In quadrilateral ABCD, AC ⊥ BD. E, F, G and H are the mid-points of AB, BC, CD and AD, respectively.

To prove: EFGH is a rectangle.

Proof:

In ΔABC, E and F are mid-points of AB and BC, respectively.

∴ EF || AC and EF $= \frac{1}{2}$AC (Mid-point theorem) ...(1)

Similarly, in ΔACD,

So, G and H are mid-points of sides CD and AD, respectively.

∴ GH || AC and GH $= \frac{1}{2}$AC (Mid-point theorem) ...(2)

From (1) and (2), we get

EF || GH and EF = GH

But this is a pair of opposite sides of the quadrilateral EFGH,

So, EFGH is parallelogram.

Now, in ΔBCD, F and G are mid-points of BC and CD, respectively.

∴ FG || BD and FG $= \frac{1}{2}$BD (Mid-point theorem) ...(3)

From (2) and (3), we get

GH || AC and FG || BD
But, AC ⊥ BD (Given)
∴ GH ⊥ FG
Hence, EFGH is a rectangle.

Answer 18:
Given: In quadrilateral ABCD, AC = BD and AC ⊥ BD. E, F, G and H are the mid-points of AB, BC, CD and AD, respectively.
To prove: EFGH is a square.
Construction: Join AC and BD.
Proof: In ΔABC,
∵ E and F are mid-points of AB and BC, respectively.
∴ EF || AC and $EF = \frac{1}{2}AC$ (Mid-point theorem) ...(1)
Similarly, in ΔACD,
∵ G and H are mid-points of sides CD and AD, respectively.
∴ GH || AC and $GH = \frac{1}{2}AC$ (Mid-point theorem) ...(2)
From (1) and (2), we get
EF || GH and EF = GH
But this a pair of opposite sides of the quadrilateral EFGH.
So, EFGH is parallelogram.
Now, in ΔBCD,
∵ F and G are mid-points of sides BC and CD, respectively.
∴ FG || BD and $FG = \frac{1}{2}BD$ (Mid-point theorem) ...(3)
From (2) and (3), we get
GH || AC and FG || BD
But, AC ⊥ BD (Given)
∴ FG ⊥ FG
But this a pair of adjacent sides of the parallelogram EFGH.
So, EFGH is a rectangle.
Again, AC = BD (Given)
⇒ $\frac{1}{2}AC = \frac{1}{2}BD$
⇒ GH = FG [From (2) and (3)]
But this a pair of adjacent sides of the rectangle EFGH.
Hence, EFGH is a square.

MULTIPLE CHOICE QUESTIONS

Answer 1:

(b) 73°

Let the measure of the fourth angle be y^o.
Since the sum of the angles of a quadrilateral is 360^o, as ,
$80^o + 95^o + 112^o + y = 360^o$
$\Rightarrow 287^o + y = 360^o$
$\Rightarrow y = 73^o$
Hence, the measure of the fourth angle is 73^o.

Answer 2:

(b) 60°

Let $\angle A = 3y$, $\angle B = 4y$, $\angle C = 5y$ and $\angle D = 6y$.
Since the sum of the angles of a quadrilateral is 360^o, as ,
$3y + 4y + 5y + 6y = 360^o$
$\Rightarrow 18y = 360^o$
$\Rightarrow y = 20^o$
$\therefore \angle A = 60^o, \angle B = 80^o, \angle C = 100^o$ and $\angle D = 120^o$

.

Answer 3:

(c) 45°

Given, $\angle BAD = 75°$ and $\angle CBD = 60°$
$\Rightarrow \angle B = 180° - \angle A\ 180° - 75° = 105°$
Thus, $\angle B = \angle ABD + \angle CBD$
$\Rightarrow 105° = \angle ABD + 60°$
$\Rightarrow \angle ABD = 105° - 60° = 45°$
$\Rightarrow \angle ABD = \angle BDC = 45°$

Answer 4:

Given, $\angle ACB = 50°$ and ∠A = 90∘ as it is rhombus Δ

In ΔBOC,
$90° + 50° + \angle OBC = 180°$
$\Rightarrow \angle OBC = 180° - (90 + 50) = 180 - 140°$
$\Rightarrow \angle OBC = 40°$
As $\angle OBC = \angle ADB$
Thus, $\angle ADB = 40°$
Hence, (a) is the correct answer.

Answer 5:

(d) Rectangle.
rectangle has diagonals of equal length.

Answer 6:

(d) rhombus
rhombus diagonals bisect each other at right angles.

Answer 7:

(a) 10 cm
Let PQRS be the rhombus.
∴ PQ = QR = RS = SP
Here, PR and QS are the diagonals of PQRS, where PR = 16 cm and QS = 12 cm.
Let the diagonals intersect each other at M.
We know that the diagonals of a rhombus are perpendicular bisectors of each

other.

∴ ΔPMQ is a right angle triangle, in which $MP = \frac{1}{2}\ PR = \frac{16}{2} = 8$ cm and $MQ = \frac{1}{2}\ QS = \frac{12}{2} = 6$ cm.

Now, $PQ^2 = MP^2 + MQ^2$ [Pythagoras theorem]

$\Rightarrow PQ^2 = (8)^2 + (6)^2$

$\Rightarrow PQ^2 = 64 + 36 = 100$

$\Rightarrow PQ = 10$ cm

Hence, the side of the rhombus is 10 cm.

Answer 8:

(b) 12 cm

Let PQRS be the rhombus.

∴ PQ = QR = RS = SP = 10 cm

Let PR and QS be the diagonals of the rhombus.

Let PR be y and QS be 16 cm and M be the intersection point of the diagonals.

We know that the diagonals of a rhombus are perpendicular bisectors of each other.

∴ ΔAOB is a right angle triangle in $MP = \frac{1}{2}\ PR = \frac{y}{2}$ and $MQ = \frac{1}{2}\ QS = \frac{16}{2} = 8$ cm.

Now, $PQ^2 = MP^2 + MQ^2$ [Pythagoras theorem]

$\Rightarrow 10^2 = (\frac{y}{2})^2 + 8^2 \Rightarrow (\frac{y}{2})^2 = 36 = 6^2 \Rightarrow y = 2\times 6 = 12$ cm

Answer 9:

Given: In rectangle PQRS, ∠MPD = 35°.

Since, ∠QPS = 90°

⇒ ∠MPQ = 90° − 35° = 55°

In ΔMPQ,

Since, MP = MQ (Diagonals of a rectangle are equal and bisect each other)

⇒ ∠MPQ = ∠MQP = 55° (Angles opposite to equal sides are equal)

Now, in ΔMSP,

55° + 55° + ∠SMP = 180° (Angle sum property of a triangle)

⇒ ∠SMP = 180° − 110°
⇒ ∠SMP = 70°
Thus, the acute angle between the diagonals is 70°.
Hence, the correct option is (b).

Answer 10:

(c) Rectangle

ABCD is parallelogram with two adjacent side
$\angle A = \angle B$ …………(given)
Then $\angle A + \angle B = 180°$
$\Rightarrow 2\angle A = 180°$
$\Rightarrow \angle A = 90°$

Others angles are equal to each others
$\Rightarrow \angle A = \angle B = \angle C = \angle D = 90°$
∴ The parallelogram is rectangle.

Answer 11:

(b) 50°
in quadrilateral ABCD, AO and BO are the bisectors of ∠C = 70° and ∠D = 30°
∠A+∠B+∠C+∠D= 360°

$\angle A + \angle B = 360° - (70 + 30)° = 260°$
$\therefore \frac{1}{2}(\angle A + \angle B) = \frac{1}{2}(260)° = 130°$
In Δ AOB,
$\angle AOB = 180° - [\frac{1}{2}(\angle A + \angle B)]$
$\Rightarrow \angle AOB = 180° - 130° = 50°$

Answer 12:

(d) 90°

Sum of any two adjacent angles of a rectangle is 180°
∴, sum of angle bisectors of two adjacent angles $= \frac{1}{2} \times 180° = 90°$

∴ Intersection angle of bisectors of two adjacent angles = $180° - 90° = 90°$

Answer 13:

(c) Rectangle
parallelograms angle bisectors enclose a rectangle

Answer 14:
Given: In quadrilateral ABCD, AS, BQ, CQ and DS are angle bisectors of angles A, B, C and D.
$\angle QPS = \angle APB$ ...(1)
In ΔAPB,
$\angle APB + \angle PAB + \angle ABP = 180°$

$\Rightarrow \angle APB = 180° - \angle PAB - \angle ABP$

$\Rightarrow \angle APB = 180 - \frac{1}{2}\angle A - \frac{1}{2}\angle B$

$\Rightarrow \angle APB = 180° - \frac{1}{2}(\angle A + \angle B)$ ...(2)

From (1) and (2),

$\angle QPS = 180° - \frac{1}{2}(\angle A + \angle B)$ ...(3)

Also, $\angle QRS = 180° - \frac{1}{2}(\angle C + \angle D)$ ...(4)

From (3) and (4), we get

$\angle QPS + \angle QRS = 360° - \frac{1}{2}(\angle A + \angle B + \angle C + \angle D)$

$= 360° - \frac{1}{2}(360°)$

$= 360° - 180°$

$= 180°$

Thus, PQRS is a quadrilateral whose opposite angles are supplementary.
Hence, (d) is the correct option.

Answer 15:

(d) parallelogram
parallelogram is formed after joining the mid points of the adjacent sides of a quadrilateral.

Answer 16:

(b) Square
Square is formed after joining the mid points of the adjacent sides of a square of the sides.

Answer 17:

(d) parallelogram.
parallelogram is formed after joining the mid points of the adjacent sides of a parallelogram i

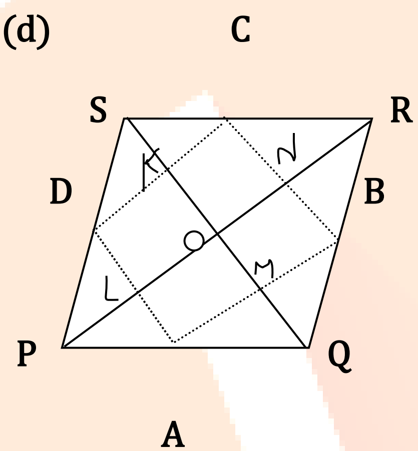
Answer 18:

(a) rhombus
Rhombus is formed after joining the mid points of the adjacent sides of a rectangle

Answer 19:

(c) Rectangle
Rectangle quadrilateral formed after joining the mid points of the adjacent sides of a rhombus.

Answer 20: (d)



ABCD is always parallelogram

By midpoint theorem,

DA∥QS and AB∥PR

⇒LA∥OM and OM∥LA ⇒ LMOA is a parallelogram.

⇒ ∠LAM= ∠LOM = 90° [PR⊥SQ (given)]

Now, ABCD is parallelogram with one angle ∠A= 90°

∴ ABCD is rectangle if PR⊥SQ

Answer 21:

Given:
The quadrilateral PQRS is a rhombus.
Thus, the sides PQ, QR, RS and SP are equal.
In ΔLMO,
$RS = \frac{1}{2}MN$ ...(1)
Also, in ΔLON,
$QR = \frac{1}{2}LN$ ...(2)
And, $QR = RS$
$\Rightarrow \frac{1}{2}MO = \frac{1}{2}LN$ [From (1) and (2)]
$\therefore, MO = LN$
Thus, the diagonals of LMNO are equal.
Hence, (c) is the correct option.

Answer 22: (d)

Square the quadrilateral formed after joining the mid points of the quadrilateral with diagonals perpendicular and equal to each other
Hence, (d) is the correct option.

Answer 23:

(c) 72°
Let PQRS is a parallelogram.
$\therefore \angle P = \angle R$ and $\angle Q = \angle S$ (Opposite angles)
Let $\angle P = y$ and $\angle Q = \frac{2}{3}y$
$\therefore \angle P + \angle Q = 180°$
$\Rightarrow y + \frac{2}{3}y = 180°$
$\Rightarrow \frac{5}{3}y = 180°$
$\Rightarrow y = 108°$
$\therefore \angle Q = \frac{2}{3} \times (108°) = 72°$
Hence, $\angle P = \angle R = 108^o$ and $\angle Q = \angle S = 72^o$

Answer 24:

(c)112°
Let PQRS is a parallelogram.
∴ ∠P = ∠R and ∠Q = ∠S
Let ∠P =y
$\therefore \angle Q = (2y - 24)°$
Now, $\angle P + \angle Q = 180°$
$\Rightarrow y + 2y - 24° = 180°$
$\Rightarrow 3y = 204°$
$\Rightarrow y = 68°$
$\therefore \angle Q = 2 \times 68° - 24° = 112°$
Hence, ∠P = ∠R = 68° and ∠Q = ∠S = 112°

Answer 25:

(c) Trapezium
Let the angles be (3y), (7y), (6y) and (4y).
Now $3y + 7y + 6y + 4y = 360°$
$\therefore y = 18°$
Thus, angles will be

$3 \times 18° = 54°$

$7 \times 18° = 126°$,

$6 \times 18° = 108°$,

$4 \times 18° = 72o°$
As, $54° + 126° = 180°$ *and* $72° + 108° = 180°$
∴ ABCD is a trapezium.

Answer 26:

(c) The opposite angles in a parallelogram are bisected by the diagonals.

Answer 27:

(c) Rectangle
It is obvious that the bisectors will enclose a rectangle.

If AMB and CND are two parallel lines, then the bisectors of $\angle AMN$, $\angle BMN$, $\angle NMP$ and $\angle NMD$ enclose a rectangle.

Answer 28:

(c) 60°

∠ABD = ∠CDB = 45° …………alternative interior angles
$\angle BAD = \angle BCD = 75°$
In Δ BCD, $\angle C = 75°$

⇒ ∠CBD+∠BCD+∠BDC = 180°
$\therefore \angle CBD = 180° - (75° + 45°) = 60°$

Answer 29:

(c) A < B
Let us assume that x be height of the parallelogram.
Now clearly, x < b
$\therefore A = a \times x < a \times b = B$
$\therefore, A < B.$

Answer 30:

(b) AF = 2 AB
In parallelogram ABCD,
$AB \,||\, DC$
$\angle DCE = \angle EBF$
In Δ DCE and Δ BFE,
$\angle DCE = \angle EBF$ (Proved above)

$\angle DEC = \angle BEF$
$BE = CE$ (Given)
By parallelogram theorem
∴, $\Delta\ DCE \cong \Delta\ BFE$
∴ $DC = BF$
Now DC= AB, since ABCD is a parallelogram.
∴ $DC = AB = BF$ ...(i)
Now, $AF = AB + BF$ ...(ii)
From (i),
∴ $AF = AB + AB = 2AB$

Answer 31:
Given: In ΔABC, R, S, D and E are the mid-points of BP, CP, AB and AC
In ΔABP,
∴ BR $= \frac{1}{2}$AP and BR || AP ...(i)
In ΔACP,
∴ ES $= \frac{1}{2}$AP and ES || AP ...(ii)
From (i) and (ii)
BR = ES and BR || ES
As BR and ES are opposite sides of the quadrilateral, thus it is a parallelogram.
Thus, (b) is the correct answer.

Answer 32:

(b) $\frac{1}{2}$(a+b)
Suppose PQRS is a trapezium.
Draw YZ parallel to PQ.
Join QS to cut YZ at X.
Now, in Δ SPQ, Y is the midpoint of PS and YX || PQ.
∴ M is the mid point of QS and YX = $\frac{1}{2}$ (a)
Similarly, M is the mid point of QS and XZ || DC.
i.e., Z is the midpoint of QR and XZ = $\frac{1}{2}$ (b)

∴ YZ = YX + XZ = $\frac{1}{2}$ (a+b)

Answer 33:

(d) $\frac{1}{2}(AB - CD)$
Join CF and produce it to cut AB at M.
Then $\Delta CDF \cong \Delta MBF$ [DF = BF, $\angle DCF = \angle BMF$ and $\angle CDF = \angle MBF$]
$\therefore CD = MB$
Thus, in ΔCAM, the points E and F are the mid points of AC and CM, respectively.
$\therefore EF = \frac{1}{2}(AM) = \frac{1}{2}(AB - MB) = \frac{1}{2}(AB - CD)$

Answer 34:

(c) 90°
$B\angle B = \angle D$
$\Rightarrow \frac{1}{2}\angle B = \frac{1}{2}\angle D$
$\Rightarrow \angle ADB = \angle ABD$
$\therefore \Delta ABD$ is an isosceles triangle and M is midpoint of BD.
$\therefore AM \perp BD$ thus, $\angle AMB = 90°$

Answer 35:

(c) $AC^2 + BD^2 = 4AB^2$
As diagonals of a rhombus bisect each other at right angles.
$\Rightarrow OA = \frac{1}{2}AC$

$OB = \frac{1}{2}BD$ and $\angle AOB = 90°$

By Pythagoras theorem , ΔAOB
Now, $(AB)^2 = (OA)^2 + (OB)^2$

$\Rightarrow \frac{1}{4}(AC)^2 + \frac{1}{4}(BD)^2$
$\therefore 4AB^2 = (AC^2 + BD^2)$

Answer 36:

(c) $BC^2 + AD^2 + 2AB.CD$
Draw perpendicular from D and C on AB which meets AB at M and N, respectively.
∴ DMNC is a parallelogram and MN = CD.
In ΔABC, ∠B is acute.
∴ $AC^2 = BC^2 + AB^2 - 2AB.AM$
In ΔABD, ∠A is acute.
∴ $BD^2 = AD^2 + AB^2 - 2AB.AN$
∴ $AC^2 + BD^2$

$= (BC^2 + AD^2) + (AB^2 + AB^2) - 2AB(AM + BN)$
$= (BC^2 + AD^2) + 2AB(AB - AM - BN)$ [AB=AM+MN+NB and AB-AM=BM]

$= (BC^2 + AD^2) + 2AB(BM - BN)$
$= (BC^2 + AD^2) + 2AB.MN$
∴ $AC^2 + BD^2 = (BC^2 + AD^2) + 2AB.CD$

Answer 37:

(d) 1:1
Area of a parallelogram = base × height
The height will be same for any pair of parallelograms with same base and same parallel lines.

Answer 38:

(b) $\frac{1}{3}AC$
Let X be the mid point of FC. Join DX.
In ΔBCF, D is the mid point of BC and X is the mid point of FC.
∴ DX || BF
⇒ DX || EF
In Δ ADX, E is the mid point of AD and EF || DX.
i.e., F is the mid point of AX.

Now, AF = FG = GC

$\therefore AF = \frac{1}{3}AC$

Answer 39: (A)

Given, $\angle AOB = 70°$

$\angle OAD = \angle OCB = 30°$ (Alternate interior angles)

As we know that Linear pair of angles is 180°

$\angle AOB + \angle BOC = 180°$

$\therefore \angle BOC = 180° - 70° = 110°$

In ΔBOC,

$\angle OBC + \angle BOC + \angle OCB = 180°$

$\angle OBC = 180° - \angle BOC - \angle OCB$

$\angle OBC = 180° - (110° + 30°) = 40°$

$\therefore \angle DBC = 40^o$

Answer 40:

(c) I and II

The statement III false, any triangle that will be formed on joining midpoints of sides of an isosceles triangle will be an isosceles triangle.

Answer 41:

(b) II and III

The statement I is not true as diagonal of rectangle does not bisect ∠A and ∠C.

SHORT ANSWER QUESTIONS

Answer 42:

Given, SR = 2cm and PR = 5cm.

As, the opposite angles of quadrilateral are equal, so PQRS is a parallelogram.
$\Rightarrow SR = PQ$
$\therefore SR = PQ = 2$ cm

Answer 43:

The parallelogram diagonals bisect each other , thus the statement is not true.

Answer 44:

Given : $\angle P + \angle S = 180°$.
i.e. the sum of the adjacent angles is equal to180°.

PQ∥RS and also $\angle R + \angle S = 180°$

Hence PQRS is a parallelogram.

Answer 45:

Acute angles is less than 90°. It is clear if all angles are less than 90°, then sum all angles will be less than 360°, thus a quadrilateral cannot be formed.

Answer 46:

It mean all angles is 90°. As rectangle and square have all angles as right angles , thus the statement holds true.

Answer 47:

It means obtuse angles is greater than 90°. It is clear if all angles are greater than 90°, then sum all angles will be greater than 360°, thus a quadrilateral cannot be formed.

Answer 48:

As the sum of all the angles given is $70° + 115° + 60° + 120° = 365°$
Thus, a quadrilateral with these angles cannot be formed.

Sum of all the angles should be exact 360°.

Answer 49:

As, the sum of all angles is equal to 360° in a quadrilateral .
Let each angle of the quadrilateral be y.

$$y + y + y + y = 360°$$

$\Rightarrow 4y = 360°$
$\Rightarrow y = 90°$
⇒ All the angles of the quadrilateral are 90°.
Thus, the given quadrilateral is a rectangle.

Answer 50:

Given, AB=7.2cm, BC= 9.8cm, AC = 3.6cm

In ΔABC,
As, D and E are the mid-points of sides AB and BC .

$DE = \frac{1}{2}(AC) = \frac{1}{2}(3.6)$

$\Rightarrow DE = 1.8$ cm
Thus, DE is equal to 1.8 cm.

Answer 51:

As the diagonals of the quadrilateral bisect each other , thus PQRS is a parallelogram. And given, $\angle Q = 56°$

Angels at liner equations ,
Thus, ∠Q+∠R=180°
⇒56°+∠R=180°

$\Rightarrow \angle R = 180° - 56°$

⇒∠R=124°

Answer 52:
Given: Parallelograms BDEF and AFDE.

F is mid point of AB, A
As, BF = DE ...(i)
And, AF = DE ...(ii)
From (i) and (ii)
AF = FB

Answer 53:

As it is clear that when the diagonals of a quadrilateral bisects each other, then it is a parallelogram and when the diagonals of a quadrilateral are equal, then it is not necessarily a parallelogram .
∴ I gives the answer and II does not give the answer.

Thus, (a) is the correct answer.

Answer 54:

It is clear that neither I alone nor II alone is sufficient to answer .
On the other hand, on considering both I and II together it will give the

answer.
∴ , (c) is the correct answer.

Answer 55:

As it is clear that when the diagonals of a parallelogram are equal, and intersect each other at right angle then the parallelogram is a square.
Thus, (c) is the correct answer.

Answer 56:

It is clear that when I or II holds true, the quadrilateral is a parallelogram.
Thus, (b) is the correct answer.

Answer 57:

(a) Both Assertion and Reason are true and Reason is a correct explanation of Assertion.
Fourth angle $= 360° - (130° + 70° + 60°) = 100°$
It is obvious that the assertion (A) and reason(R) is absolutely true.
On the same hand the reason (R) can be proved easily. Thus, (R) is true as well.

As, the reason (R) hold the assertion (A).

Thus, (a) is the correct answer .

Answer 58:

(a) Both Assertion and Reason are true and Reason is a correct explanation of Assertion.
It is obvious that the assertion (A) and reason(R) is absolutely true.

On the same hand the reason (R) can be proved easily. Thus, (R) is true as well.

As, the reason (R) hold the assertion (A).

Thus, (a) is the correct answer .

Answer 59:

(b) Both Assertion and Reason are true but Reason is not a correct explanation of Assertion.

It is obvious that the assertion (A) is absolutely true.
On the same hand the reason (R) can be proved easily. Thus, (R) is true as well.

As, the reason (R) does not hold the assertion (A).

Thus, (b) is the correct answer .

Answer 60:

(d) Assertion is false and Reason is true.

It is obvious that the assertion (A) is absolutely false.
On the same hand the reason (R) can be proved easily. Thus, (R) is true as well.

Thus, (d) is the correct answer.

Answer 61:

(b) Both Assertion and Reason are true but Reason is not a correct explanation of Assertion.
It is obvious that the assertion (A) is absolutely true.

On the same hand the reason (R) can be proved easily. Thus, (R) is true as well.

As, the reason (R) does not hold the assertion (A).

Thus, (b) is the correct answer .

Answer 62:

(a) will go with (q),

(b) will go with (r),

(c) will go with (s),

(d) will go with (p)

Answer 63:

(a) - (r), (b) - (s), (c) - (p), (d) - (q)

(a) $PQ = \frac{1}{2}(AB + CD) = \frac{1}{2}(17) = 8.5$ cm

(b) $OR = \frac{1}{2}(PR) = \frac{1}{2}(13) = 6.5$ cm