Radiation reactions in velocity Verlet pusher

Severin Diederichs

June 11, 2023

1 Introduction

This document briefly describes how the radiation reactions (RR) are implemented in the beam particle pusher in https://github.com/Hi-PACE/hipace. The beam particle pusher is a velocity Verlet pusher. RR have been implemented in various codes and cover various regimes, from classical electrodynamics to small quantum corrections to full quantum treatment. A good explanation is given e.g., in the Smilei documentation. Here, we follow the implementation of RR in the classical regime as proposed for the Boris pusher in Tamburini et al. [2010]. In this document, we derive it analogously for the velocity Verlet pusher.

In the classical regime, the RR can be described as a drag term in the force:

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = \mathbf{f}_L - d\mathbf{v} \tag{1}$$

with $\mathbf{f}_L = -(\mathbf{E} + \mathbf{v} \times \mathbf{B})$ being the Lorentz force and the drag term d given by

$$d = \left(\frac{4}{3}\pi\frac{r_e}{\lambda}\right)\gamma^2\left[\mathbf{f}_L^2 - (\mathbf{v}\cdot\mathbf{f}_L)^2\right] \tag{2}$$

with r_e being the classical electron radius, λ being the laser wavelength (note that this will need to be fixed), and γ the Lorentz factor.

The total force $\mathbf{f}$ can be written as

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} = \mathbf{f}_L + \mathbf{f}_R \tag{3}$$

with

$$\mathbf{f}_R = -\left(\frac{4}{3}\pi\frac{r_e}{\lambda}\right)\left\{\mathbf{f}_L \times \mathbf{B} - (\mathbf{v}\cdot\mathbf{E})\mathbf{E} + \gamma^2\left[\mathbf{f}_L^2 - (\mathbf{v}\cdot\mathbf{E})^2\right]\mathbf{v}\right\} \tag{4}$$

2 Numerical implementation with velocity Verlet pusher

The momentum is updated in the velocity Verlet pusher with

$$\frac{\mathbf{p}^{(n+1)} - \mathbf{p}^{(n)}}{\Delta t} = \mathbf{f}^{(n+1/2)} = \mathbf{f}_L^{(n+1/2)} + \mathbf{f}_R^{(n+1/2)} \tag{5}$$

with Δt being the time step. The momenta and forces are known at integer and integer half steps, respectively (note that this differs from the Boris pusher, where it is vice-versa).

Now, we consider two helper momenta $\mathbf{p}_L$ and $\mathbf{p}_R$:

$$\frac{\mathbf{p}_L^{(n+1)} - \mathbf{p}_L^{(n)}}{\Delta t} = \mathbf{f}_L^{(n+1/2)} \tag{6}$$

$$\frac{\mathbf{p}_R^{(n+1)} - \mathbf{p}_R^{(n)}}{\Delta t} = \mathbf{f}_R^{(n+1/2)} \tag{7}$$

and we further assume that $\mathbf{p}_L^{(n)} = \mathbf{p}_R^{(n)} = \mathbf{p}^{(n)}$. With that assumption, we obtain

$$\mathbf{p}^{(n+1)} = \mathbf{p}_L^{(n+1)} + \mathbf{p}_R^{(n+1)} - \mathbf{p}^{(n)} \tag{8}$$

Using equation 7, and 8, we obtain

$$\mathbf{p}^{(n+1)} = \mathbf{p}_L^{(n+1)} + \mathbf{f}_R^{(n+1/2)}\Delta t \tag{9}$$

Now, to compute the Lorentz and RR force terms, we need to estimate the velocity of the electron at the intermediate step $n+1/2$. To achieve this, we advance $\mathbf{p}^{(n)}$ to $\mathbf{p}_L^{(n+1)}$ using the Lorentz force, and then we use this to estimate the *total* momentum $\mathbf{p}^{(n+1/2)}$ and velocity $\mathbf{v}^{(n+1/2)}$ at the intermediate half step as

$$\mathbf{p}^{(n+1/2)} \approx \frac{\mathbf{p}_L^{(n+1)} + \mathbf{p}^{(n)}}{2} \tag{10}$$

and

$$\mathbf{v}^{(n+1/2)} = \frac{\mathbf{p}^{(n+1/2)}}{\gamma^{(n+1/2)}} = \frac{\mathbf{p}^{(n+1/2)}}{\sqrt{1 + \left(\mathbf{p}^{(n+1/2)}\right)^2}} \tag{11}$$

Then, we can use equation 9 to calculate the final momentum at the next full integer step $(n+1)$. Thus, the algorithm works as follows:

1. Just as in the normal velocity Verlet pusher, the particles are pushed to to $x^{n+1/2}$, the fields are gathered at $(n+1/2)$, and the momentum is updated to $\mathbf{p}_L^{(n+1)}$ with the Lorentz force.
2. The RR force $\mathbf{f}_R^{(n+1/2)}$ is calculated using equation 4 and 11, and the fields at the intermediate step.
3. The final new momentum $\mathbf{p}^{(n+1)}$ via equation 9.

References

M Tamburini, F Pegoraro, A Di Piazza, C H Keitel, and A Macchi. Radiation reaction effects on radiation pressure acceleration. *New Journal of Physics*, 12(12):123005, dec 2010. doi: 10.1088/1367-2630/12/12/123005. URL https://dx.doi.org/10.1088/1367-2630/12/12/123005.