Magnetic Field Calculation for non-symmetric setups in Kassiopeia

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PRiSMA+ Cluster of Excellence
Precision Physics, Fundamental Interactions and Structure of Matter

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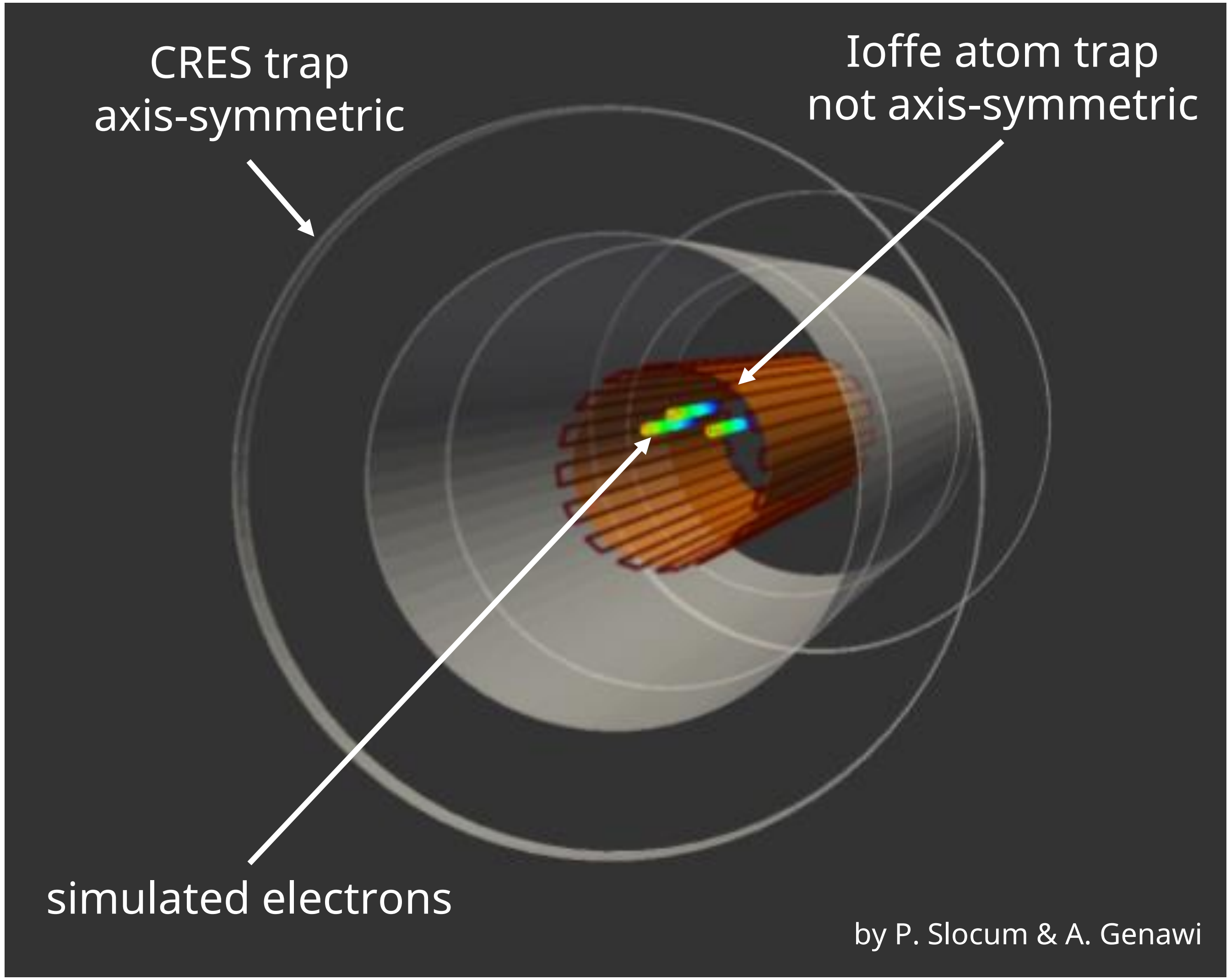
Project 8's needs

- Project 8 aims to trap CRES electrons in same volume as tritium atoms
- Atom trapping need high field walls from Ioffe or Halbach array magnets
- Ioffe and Halbach array magnets are not axis-symmetric
- Electron simulation performance is limited by magnetic field evaluation
- Simulation time scales with number of Ioffe segments (Ioffe trap polarity)



Magnetic field support in Kassiopeia

- Direct calculation
  - Numerical integration of Biot-Savart law
  - Speed depends on number of wire segments in geometry
- Zonal harmonic expansion

$$\Phi_{cen}(r_{cyl}, z_{cyl}) = \sum_{n=0}^{\infty} \Phi_n^{cen}|_{z0} \left(\frac{\rho}{\rho_{cen}}\right)^n P_n(\cos\theta)$$

  - Only valid for axis-symmetric problems
  - Precomputes coefficients before simulation
  - Speed depends on number of expansion coefficients
- Interpolation from field maps
  - Field maps in VTI format on rectangular grid
  - Gradients can be calculated numerical or can be provided in VTI file
  - Interpolation between grid points: linear, cubic
- Superposition
  - Several solver solution can be superimposed
  - Constant magnetic field

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only way for
non-symmetric setups

used for symmetric setups

interpolation does not obey
Maxwell's equations
→ can lead to energy loss

files / memory get huge for
fine grid

used for background fields

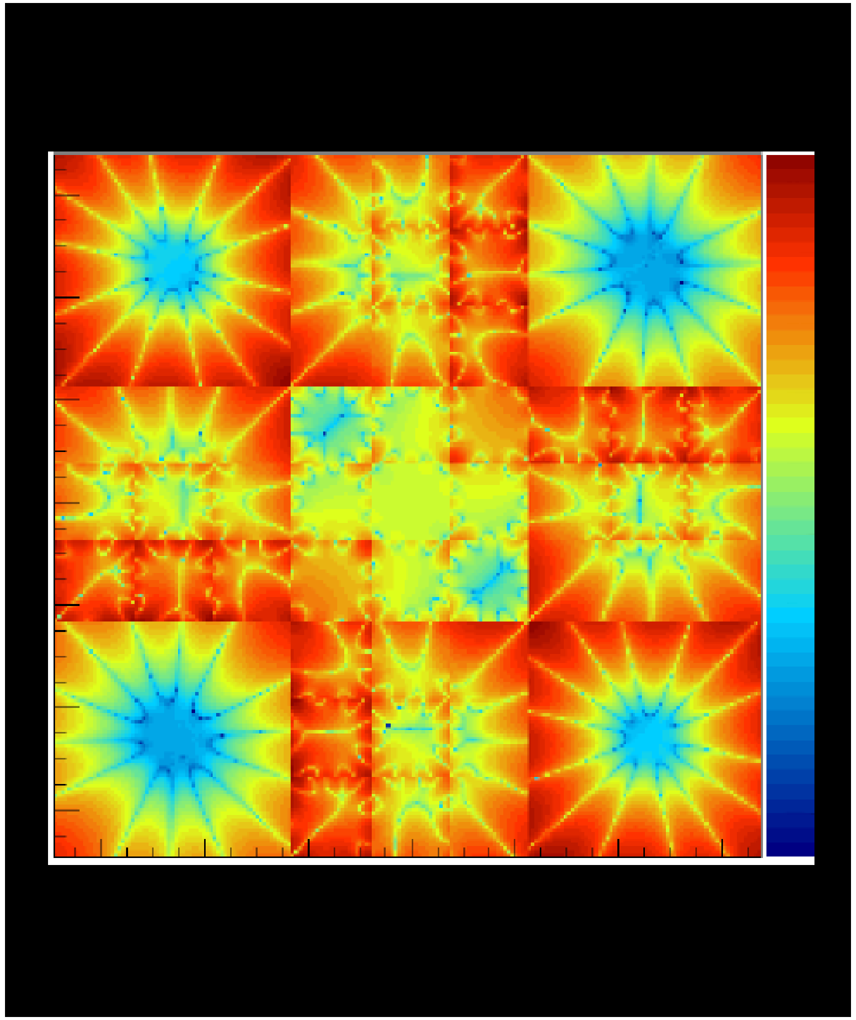
Proposed solutions

- Fast Fourier Transform on Multipoles
- Upgrade ZonalHarmonic solver to SphericalHarmonic solver
- Import of unsorted grids

Fast Fourier Transform on Multipoles

- Fast evaluation of solutions for Laplace equation
- Calculates E-field on surface & treats the flux as “surface charge”
- Segments space in tree structure (fast lookup)
- In each tree leave stores multipole field from “surface charges” far away and close by surface charges (few terms to evaluate)
- Implemented for E-fields in Kassiopeia (John Barret, MIT, PhD thesis for KATRIN)
- Magnetic fields in current free space are described by same physics equations
- Replace “surface charge” by “flux through surface”

- Copy & past from E-field implementation
- Described in GitHub Issue 59 [Enhancement] https://github.com/KATRIN-Experiment/Kassiopeia/issues/59



ZonalHarmonic → SphericalHarmonic

- Spherical harmonics are general solution to Laplace equation in spherical coordinates

$$\phi(r,\theta,\phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{m=+l} \left(a_{lm} r^l + \frac{b_{lm}}{r^{l+1}} \right) Y_{lm}(\theta,\phi)$$

- Definition of spherical harmonics

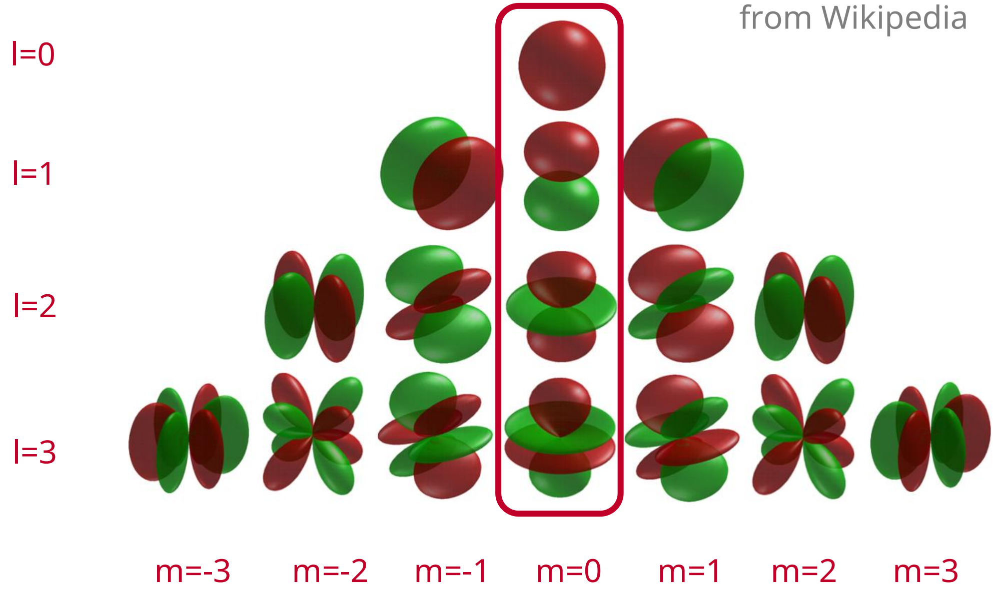
$$Y_{lm}(\theta,\phi) = \sqrt{\frac{2l+1}{4\pi}\frac{(l-m)!}{(l+m)!}} P_l^m(\cos\theta)\exp(im\phi)$$

- Terms with m=0 are cylinder symmetric

$$\phi(r,\theta) = \sum_{l=0}^{\infty} \left(a_l r^l + \frac{b_l}{r^{l+1}} \right) P_l(\cos\theta)$$

- ZonalHarmonics implements special case for *m=0*

from Wikipedia

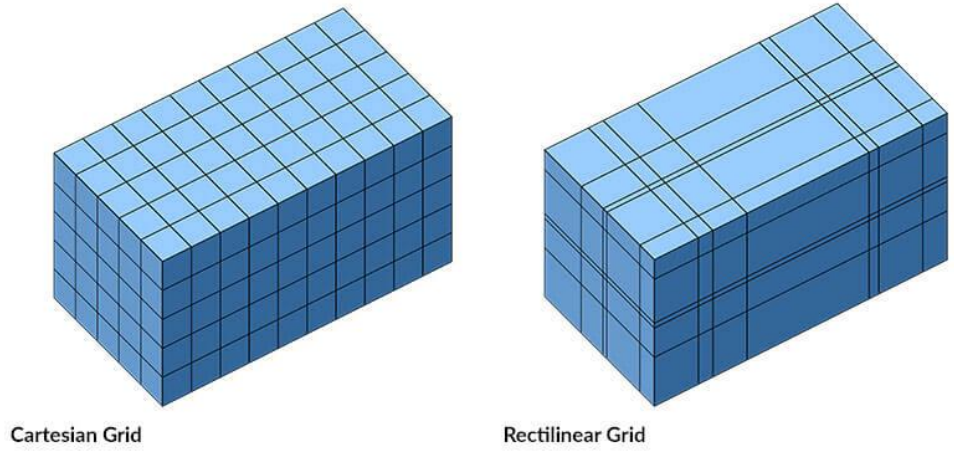


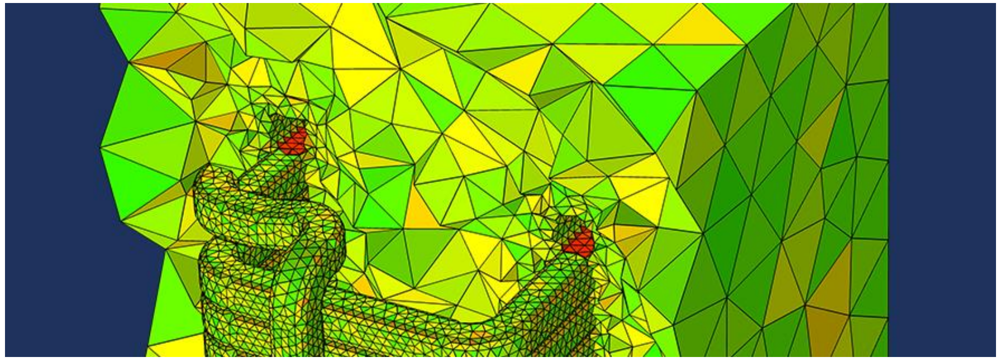
- Implementation ideas
  - a) full spherical harmonic solver
  - b) read in expansion coefficients from config file / separate file and only evaluate

Import of unstructured grids

- VTI format allows only cartesian grids (at max rectilinear grids)
- To account for strong gradients in small region of space, full space has to be segmented in very fine grid
- Solvers like COMSOL generate unstructured grids that are optimized to gradients in the problem
- 3D grid consists of tetrahedral
- Implement import from unstructured grids (e.g. COMSOL)
- Use interpolation in tetrahedral using barycentric coordinate system

from PENtrack





from COMSOL

Summary

- No fast solver for non-symmetric magnetic fields

- Proposed solutions
  - Fast Fourier Transform on Multipoles (FFTM)
  - Upgrade ZonalHarmonic solver to SphericalHarmonic solver
  - Import of unsorted grids

- Project 8 works on implementing the above solutions
  - priority on FFTM

- Help or additional advice is very welcome