**31.4.15. Ranked Probability Score (RPS)**

While the above probabilistic verification measures utilize dichotomous observations, the Ranked Probability Score (RPS, Epstein 1969, Murphy 1969) is the only probabilistic verification measure for discrete multiple-category events available in MET. It is assumed that the categories are ordinal as nominal categorical variables can be collapsed into sequences of binary predictands, which can in turn be evaluated with the above measures for dichotomous variables (Wilks 2011). The RPS is the multi-category extension of the Brier score (Tödter and Ahrens 2012), and is a proper score (Mason 2008).

Let $J$ be the number of categories, then both the forecast, $f=\left(f\_{1},…,f\_{J}\right)$, and observation, $o=\left(o\_{1},…,o\_{J}\right)$, are length-$J$ vectors, where the components of $f$ include the probabilities forecast for each category $1,…,J$ and $o$ contains 1 in the category that is realized and zero everywhere else. The cumulative forecasts, $F\_{m}$, and observations, $O\_{m}$, are defined to be:

$F\_{m}=\sum\_{j=1}^{m}f\_{j}and O\_{m}=\sum\_{j=1}^{m}o\_{j}, m=1,…,J$.

To clarify, $F\_{1}=f\_{1}$ is the first component of $F\_{m}$, $F\_{2}=f\_{1}+f\_{2}$, etc., and $F\_{J}=1$. Similarly, if $o\_{j}=1$ and $i<j$, then $O\_{i}=0$ and when $i\geq j$, $O\_{i}=1$, and of course, $O\_{J}=1$. Finally, the RPS is defined to be:

$$RPS=\sum\_{m=1}^{J}\left(F\_{m}-O\_{m}\right)^{2}=\sum\_{m=1}^{J}BS\_{m},$$

where $BS\_{m}$ is the Brier score for the $m$-th category (Tödter and Ahrens 2012). Subsequently, the RPS lends itself to a decomposition into reliability, resolution and uncertainty components, noting that each component is aggregated over the different categories; these are the line types RPS\_REL, RPS\_RES and RPS\_UNC.

### References used here that are not currently in the MET document:

Epstein, E. S., 1969. A scoring system for probability forecasts of ranked categories. *J. Appl. Meteor*., **8**, 985–987, 10.1175/1520-0450(1969)008<0985:ASSFPF>2.0.CO;2.

Mason, S. J., 2008. Understanding forecast verification statistics. *Meteor. Appl*., **15**, 31–40, doi: 10.1002/met.51.

Murphy, A. H., 1969. On the ranked probability score. *Journal of Applied Meteorology and Climatology*, **8** (6), 988 – 989, doi: 10.1175/1520-0450(1969)008<0988:OTPS>2.0.CO;2.

Tödter, J. and B. Ahrens, 2012. Generalization of the Ignorance Score: Continuous ranked version and its decomposition. *Mon. Wea. Rev.*, **140** (6), 2005 – 2017, doi: 10.1175/MWR-D-11-00266.1.