

# SUPPLEMENTARY MATERIALS: REDUCING PARALLEL COMMUNICATION IN ALGEBRAIC MULTIGRID THROUGH SPARSIFICATION\*

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The drop tolerances used for the various runs are listed below in the format  $[\gamma_1, \gamma_2, \dots]$  where  $\gamma_1$  is the drop tolerance used on the first coarse level,  $\gamma_2$  on the second coarse level, and so on. The last drop tolerance in the list is used on all remaining levels. For example:

[0.1]	0.1 on all levels
[0.0, 0.1]	0.0 on first coarse level, 0.1 on all other levels
[0.0, 0.1, 1.0]	0.0 on first coarse level, 0.1 on second coarse level, 1.0 on all remaining levels

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processors	method	$\gamma$ list
128	NonGalerkin	[1.0]
	Sparse (Full)	[1.0]
	Hybrid (Full)	[1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]
1024	NonGalerkin	[1.0]
	Sparse (Full)	[1.0]
	Hybrid (Full)	[1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]
4096	NonGalerkin	[1.0]
	Sparse (Full)	[1.0]
	Hybrid (Full)	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]
8192	NonGalerkin	[0.0, 0.01, 0.1, 1.0]
	Sparse (Full)	[1.0]
	Hybrid (Full)	[1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]
16384	NonGalerkin	[0.0, 0.01, 0.1, 1.0]
	Sparse (Full)	[1.0]
	Hybrid (Full)	[1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]
32768	NonGalerkin	[0.0, 0.01, 0.1, 1.0]
	Sparse (Full)	[1.0]
	Hybrid (Full)	[1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]
100000	NonGalerkin	[0.0, 0.01, 0.1, 1.0]
	Sparse (Full)	[1.0]
	Hybrid (Full)	[1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]

Table SM1: 27-Point Laplacian, Weak Scaling Studies, and the “8192 Processes” cases correspond to Figure 8

processors	method	$\gamma$ list
128	NonGalerkin	[1.0]
	Sparse (Full)	[1.0]
	Hybrid (Full)	[1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]
1024	NonGalerkin	[1.0]
	Sparse (Full)	[1.0]
	Hybrid (Full)	[1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]
4096	NonGalerkin	[1.0]
	Sparse (Full)	[1.0]
	Hybrid (Full)	[1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]
8192	NonGalerkin	[1.0]
	Sparse (Full)	[1.0]
	Hybrid (Full)	[1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]
16384	NonGalerkin	[1.0]
	Sparse (Full)	[1.0]
	Hybrid (Full)	[1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]
32768	NonGalerkin	[1.0]
	Sparse (Full)	[1.0]
	Hybrid (Full)	[1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]
100000	NonGalerkin	[1.0]
	Sparse (Full)	[1.0]
	Hybrid (Full)	[1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]

Table SM2: 27-Point Laplacian, Strong Scaling Studies

processors	method	$\gamma$ list
128	NonGalerkin	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Full)	[0.0, 0.1]
	Hybrid (Full)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
1024	NonGalerkin	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Full)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Full)	[0.0, 0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
4096	NonGalerkin	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Full)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Full)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
8192	NonGalerkin	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Full)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Full)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
16384	NonGalerkin	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Full)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Full)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
32768	NonGalerkin	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Full)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Full)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
100000	NonGalerkin	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Full)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Full)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]

Table SM3: Rotated Anisotropic Diffusion, Weak Scaling Studies, and the “8192 Processes” cases correspond to Figure 8

processors	method	$\gamma$ list
128	NonGalerkin	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Full)	[0.0, 0.01, 0.1, 1.0]
	Hybrid (Full)	[0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
1024	NonGalerkin	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Full)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Full)	[0.0, 0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
4096	NonGalerkin	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Full)	[0.0, 0.1]
	Hybrid (Full)	[0.0, 0.01, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
8192	NonGalerkin	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Full)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Full)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
16384	NonGalerkin	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Full)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Full)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
32768	NonGalerkin	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Full)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Full)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
100000	NonGalerkin	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Full)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Full)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]

Table SM4: Rotated Anisotropic Diffusion, Strong Scaling Studies

processors	method	$\gamma$ list
32	NonGalerkin	[0.0, 0.0, 0.0, 0.1]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
64	NonGalerkin	[0.01]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]
128	NonGalerkin	[0.0, 0.0, 0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
256	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]

Table SM5: Florida Sparse Matrix Collection: `2cubes_sphere.mtx`

processors	method	$\gamma$ list
32	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.01, 0.1]
	Hybrid (Diag)	[0.0, 0.01, 0.1]
64	NonGalerkin	[0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.01, 0.1, 1.0]
128	NonGalerkin	[0.0, 0.1]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.01, 0.1, 1.0]
256	NonGalerkin	[0.0, 0.01, 0.1]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.01, 0.1, 1.0]

Table SM6: Florida Sparse Matrix Collection: `G2_circuit.mtx`

processors	method	$\gamma$ list
32	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.01, 0.1, 1.0]
64	NonGalerkin	[0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
128	NonGalerkin	[0.0, 0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
256	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.01, 0.1]

Table SM7: Florida Sparse Matrix Collection: `G3_circuit.mtx`



processors	method	$\gamma$ list
32	NonGalerkin	[0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
64	NonGalerkin	[0.0, 0.01, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
128	NonGalerkin	[0.0, 0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
256	NonGalerkin	[0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.01, 0.1, 1.0]

Table SM8: Florida Sparse Matrix Collection: `af_0_k101.mtx`

processors	method	$\gamma$ list
32	NonGalerkin	[0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
64	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
128	NonGalerkin	[0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
256	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.01, 0.1, 1.0]

Table SM9: Florida Sparse Matrix Collection: `af_1_k101.mtx`

processors	method	$\gamma$ list
32	NonGalerkin	[0.01]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
64	NonGalerkin	[0.0, 0.0, 0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
128	NonGalerkin	[0.0, 0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
256	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.01, 0.1, 1.0]

 Table SM10: Florida Sparse Matrix Collection: `af_2_k101.mtx`

processors	method	$\gamma$ list
32	NonGalerkin	[0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
64	NonGalerkin	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
128	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.01, 0.1, 1.0]
256	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.01, 0.1, 1.0]

Table SM11: Florida Sparse Matrix Collection: `af_3_k101.mtx`

processors	method	$\gamma$ list
32	NonGalerkin	[0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
64	NonGalerkin	[0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
128	NonGalerkin	[0.0, 0.01, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
256	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.01, 0.1, 1.0]

 Table SM12: Florida Sparse Matrix Collection: `af_4_k101.mtx`

processors	method	$\gamma$ list
32	NonGalerkin	[0.0, 0.01, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
64	NonGalerkin	[0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
128	NonGalerkin	[0.01]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]
256	NonGalerkin	[0.0, 0.1]
	Sparse (Diag)	[0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.01, 0.1]

Table SM13: Florida Sparse Matrix Collection: `af_5_k101.mtx`

processors	method	$\gamma$ list
32	NonGalerkin	[0.01]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
64	NonGalerkin	[1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
128	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
256	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]

 Table SM14: Florida Sparse Matrix Collection: `af_shell13.mtx`

processors	method	$\gamma$ list
32	NonGalerkin	[0.01]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
64	NonGalerkin	[1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
128	NonGalerkin	[0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
256	NonGalerkin	[0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]

Table SM15: Florida Sparse Matrix Collection: `af_shell14.mtx`



processors	method	$\gamma$ list
32	NonGalerkin	[0.01]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
64	NonGalerkin	[1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
128	NonGalerkin	[0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
256	NonGalerkin	[0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]

Table SM16: Florida Sparse Matrix Collection: `af_shell17.mtx`

processors	method	$\gamma$ list
32	NonGalerkin	[0.01]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
64	NonGalerkin	[1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
128	NonGalerkin	[0.0, 0.01, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
256	NonGalerkin	[0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]

Table SM17: Florida Sparse Matrix Collection: `af_shell18.mtx`

processors	method	$\gamma$ list
32	NonGalerkin	[0.0, 0.01, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
64	NonGalerkin	[0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
128	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
256	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]

 Table SM18: Florida Sparse Matrix Collection: `apache2.mtx`

processors	method	$\gamma$ list
32	NonGalerkin	[0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
64	NonGalerkin	[0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
128	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.01, 0.1, 1.0]
256	NonGalerkin	[0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]

Table SM19: Florida Sparse Matrix Collection: `ecology2.mtx`

processors	method	$\gamma$ list
32	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]
64	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
128	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
256	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.1]

 Table SM20: Florida Sparse Matrix Collection: `parabolic_fem.mtx`

processors	method	$\gamma$ list
32	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
64	NonGalerkin	[0.0, 0.0, 0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
128	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.1]
256	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]

Table SM21: Florida Sparse Matrix Collection: `thermal2.mtx`

processors	method	$\gamma$ list
32	NonGalerkin	[0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
64	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.01, 0.1, 1.0]
128	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.1]
256	NonGalerkin	[0.0, 1.0]
	Sparse (Diag)	[0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.01, 0.1, 1.0]

Table SM22: Florida Sparse Matrix Collection: `thermomech_TK.mtx`

processors	method	$\gamma$ list
32	NonGalerkin	[0.0, 0.01, 0.1, 1.0]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
	Hybrid (Diag)	[0.0, 0.0, 0.0, 0.01, 0.1, 1.0]
64	NonGalerkin	[0.0, 0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
128	NonGalerkin	[0.0, 0.0, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]
256	NonGalerkin	[0.0, 0.01, 0.1]
	Sparse (Diag)	[0.0, 0.0, 0.1]
	Hybrid (Diag)	[0.0, 0.0, 0.1]

Table SM23: Florida Sparse Matrix Collection: `tmt_sym.mtx`