Introducing Global Phase Gate in Qiskit Terra

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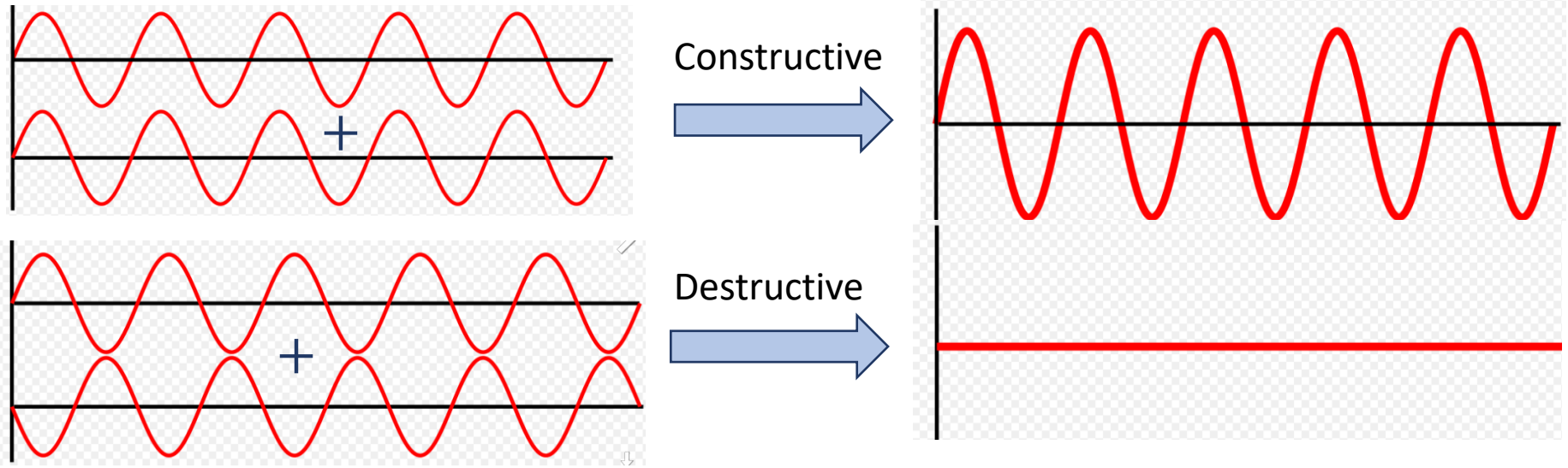
What is a phase?

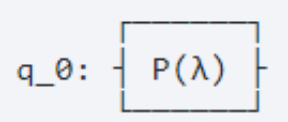
$$e^{i\phi} = \cos\phi + i\sin\phi$$

Local (Relative) Phase	Global Phase
$\|\psi\rangle = \cos\left(\frac{\theta}{2}\right)\|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)\|1\rangle$	$\|\varphi\rangle = e^{i\omega_o}\left\{\cos\left(\frac{\theta}{2}\right)\|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)\|1\rangle\right\}$
Complex (relative) amplitude (Does not affect probabilities)	Complex overall amplitude
Resource for interference & Entanglement	Not physically relevant



Examples of (relative) phase gates

- Currently available 1-qubit (relative) phase gates in Qiskit:
  - Pauli-Z gate ($ZGate$): Relative phase of π $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
  - $SGate$: Relative phase of $\frac{\pi}{2}$ or $\sqrt{ZGate}$ $S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$
  - $TGate$: Relative phase of $\frac{\pi}{4}$ or $\sqrt[4]{ZGate}$ $T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$
- Generalized 1-qubit (relative) phase gate
  - $PhaseGate(\lambda)$: Applies relative phase of λ.



$$P(\lambda) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\lambda} \end{pmatrix}$$

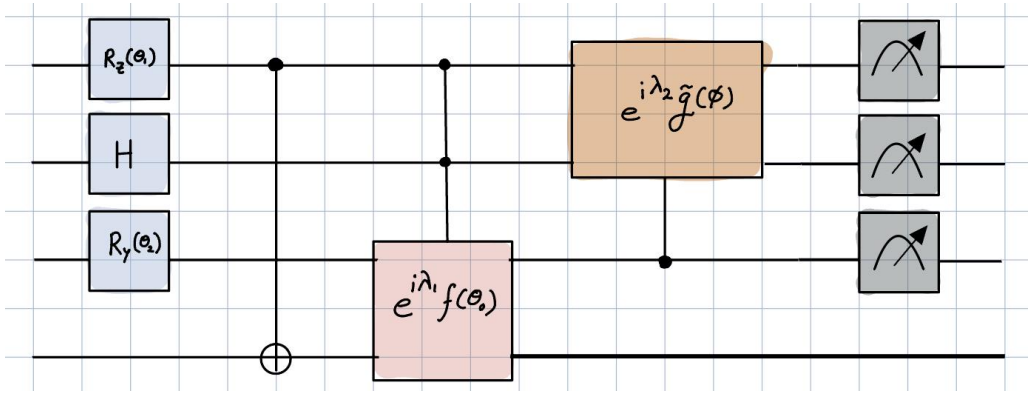
$$P(\lambda = \pi) = Z$$

$$P(\lambda = \pi/2) = S$$

$$P(\lambda = \pi/4) = T$$

Why Global Phase?

- Appending a global phase in front of a sub-part of the quantum circuit.



- Controlled version of the sub-circuit with the global phase.
- Currently there exists a way to set a global phase for a quantum circuit
  - **global_phase** attribute of the **QuantumCircuit** class.
  - But it is quite clunky and not user friendly.

The GlobalPhaseGate

```
class GlobalPhaseGate(Gate):
    r"""The global phase gate (:math:`e^{i\theta}`).

    Can be applied to a :class:`~qiskit.circuit.QuantumCircuit`

    **Mathamatical Representation:**


    .. math::
        \text{GlobalPhaseGate}\ =
            \begin{pmatrix}
                e^{i\theta}
            \end{pmatrix}

    **Circuit symbol:**

    .. parsed-literal::
    """
```

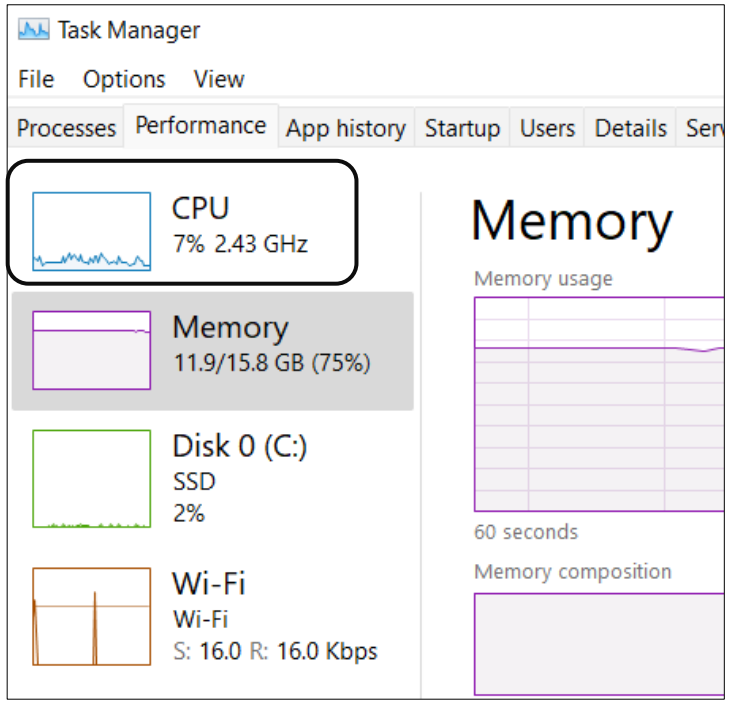
```
def __init__(self, phase: ParameterValueType, label: Optional[str] = None):
    """Create new globalphase gate.
    Args:
        phase: The value of phase it takes.
        label: An optional label for the gate.
    """
    super().__init__("global_phase", 0, [phase], label=label)
```

Testing the proposed changes

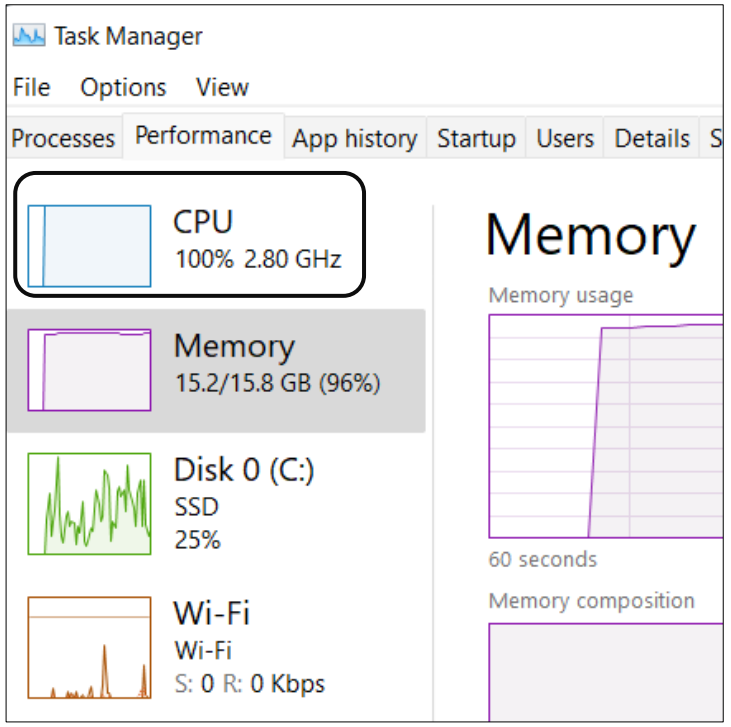
```
ing_extensions==4.4.0,urllib3==1.26.12,voluptuous==0.13.1,wcwidth==
nt==1.4.1,widgetsnbextension==4.0.3,wrapt==1.12.1,zipp==3.10.0
black run-test-pre: PYTHONHASHSEED='321'
black run-test: commands[0] | black qiskit test tools examples setu
All done! ✨ 🍰 ✨
1635 files left unchanged.
_____________________________________ summary _____________________________________
  black: commands succeeded
  congratulations :)

(qiskit-dev) C:\Users\sumit\Desktop\GitProjects\qiskit-terra>
```

```
======
Totals
======
Ran: 14181 tests in 533.3786 sec.
 - Passed: 13871
 - Skipped: 306
 - Expected Fail: 4
 - Unexpected Success: 0
 - Failed: 0
Sum of execute time for each test: 6058.6479 sec.

==============
Worker Balance
==============
 - Worker 0 (1160 tests) => 0:08:45.105007
 - Worker 1 (1163 tests) => 0:08:48.113993
 - Worker 2 (1167 tests) => 0:08:19.059995
 - Worker 3 (1172 tests) => 0:08:38.619999
 - Worker 4 (1173 tests) => 0:07:28.985469
 - Worker 5 (1173 tests) => 0:08:32.836996
 - Worker 6 (1173 tests) => 0:08:08.891990
 - Worker 7 (1173 tests) => 0:08:53.346578
 - Worker 8 (1308 tests) => 0:08:47.915989
 - Worker 9 (1173 tests) => 0:08:16.815996
 - Worker 10 (1173 tests) => 0:08:32.290994
 - Worker 11 (1173 tests) => 0:08:12.031992

  py: commands succeeded
  congratulations :)
```

Resource Intensive Tests



Normal resource usage



Resource usage during tox tests

Pull Request

Objective

$CPhaseGate(\lambda)$:	Global Phase Gate	Controlled Global Phase Gate
Args: Phase (λ), Control Qubits, Target State	Args: Phase (θ), Quantum Circuit	Args: Phase (λ), Control Qubits, Sub-Quantum Circuit, Target State
Diagonal Symmetric Matrix	Scalar	Block diagonal Matrix
$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\lambda} \end{bmatrix}$	$e^{i\theta}U$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{i\lambda}U & \\ 0 & 0 & & \end{bmatrix}$
Phase on a single qubit	Scalar multiplication on States	Phase on a set of qubits

Prepared flexible Global Phase gate which improves the user experience.

Acknowledgement

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QAMP Fall 2022 *Team*

Thank You All!