Introducing Global Phase Gate in Qiskit Terra

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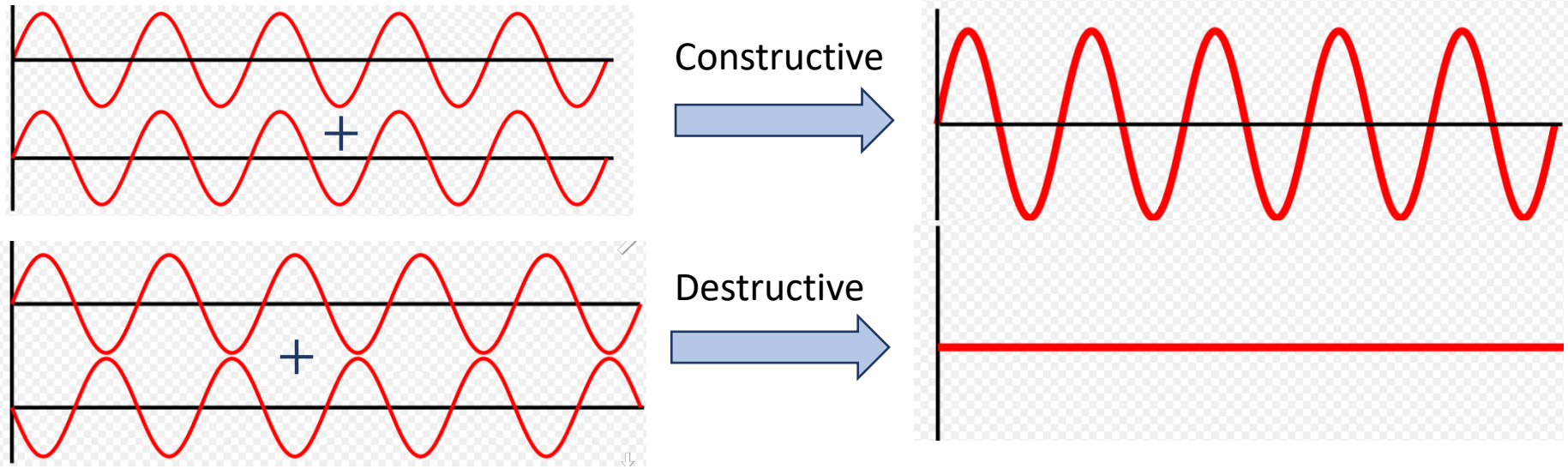
What is a phase?

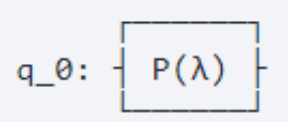
$$e^{i\phi} = \cos\phi + i\sin\phi$$

Local (Relative) Phase	Global Phase
$\|\psi\rangle = \cos\left(\frac{\theta}{2}\right)\|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)\|1\rangle$	$\|\varphi\rangle = e^{i\omega_o}\left\{\cos\left(\frac{\theta}{2}\right)\|0\rangle + e^{i\phi}\sin\left(\frac{\theta}{2}\right)\|1\rangle\right\}$
Complex (relative) amplitude (Does not affect probabilities)	Complex overall amplitude
Resource for interference & Entanglement	Not physically relevant



Examples of (relative) phase gates

- Currently available 1-qubit (relative) phase gates in Qiskit:
  - Pauli-Z gate ($ZGate$): Relative phase of π $Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
  - $SGate$: Relative phase of $\frac{\pi}{2}$ or $\sqrt{ZGate}$ $S = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$
  - $TGate$: Relative phase of $\frac{\pi}{4}$ or $\sqrt[4]{ZGate}$ $T = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$
- Generalized 1-qubit (relative) phase gate
  - $PhaseGate(\lambda)$: Applies relative phase of λ.



$$P(\lambda) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\lambda} \end{pmatrix}$$

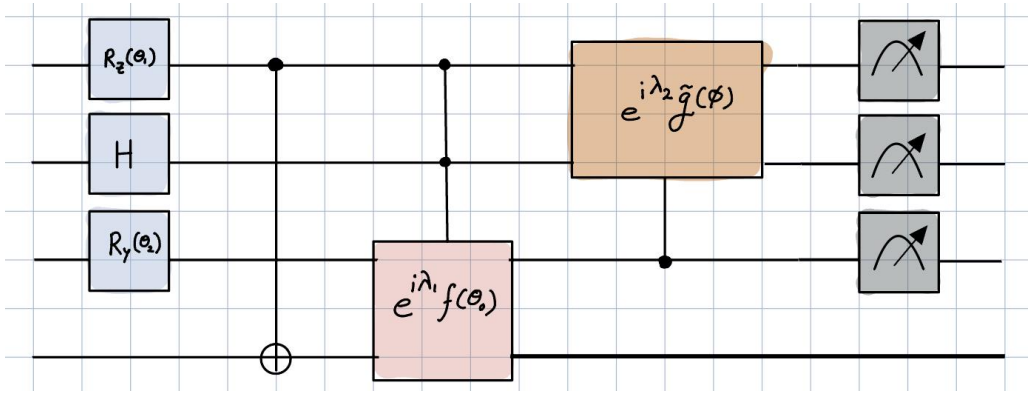
$$P(\lambda = \pi) = Z$$

$$P(\lambda = \pi/2) = S$$

$$P(\lambda = \pi/4) = T$$

Why Global Phase?

- Appending a global phase in front of a sub-part of the quantum circuit.



- Controlled version of the sub-circuit with the global phase.
- Currently there exists a way to set a global phase for a quantum circuit
  - **global_phase** attribute of the **QuantumCircuit** class.
  - But it is quite clunky and not user friendly.

Objective

$CPhaseGate(\lambda)$:	Global Phase Gate	Controlled Global Phase Gate
Args: Phase (λ), Control Qubits, Target State	Args: Phase (λ), Quantum Circuit	Args: Phase (λ), Control Qubits, Sub-Quantum Circuit, Target State
Diagonal Symmetric Matrix	Scalar	Block diagonal Matrix
$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\lambda} \end{bmatrix}$	$e^{i\lambda}U$	$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & e^{i\lambda}U & \\ 0 & 0 & & \end{bmatrix}$
Phase on a single qubit	Scalar multiplication on States	Phase on a set of qubits

Prepare flexible Global Phase gate which improves the user experience.